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AN ADAPTIVE COOPERATIVE EQUALISATION CONTROL METHOD FOR DUAL-ARM ROBOTS CONSIDERING JOINT ANGULAR ACCELERATION CONSTRAINTS

Guichao Cai,* Yutong Zeng,* Yuzhang She,* Haocong Zheng,* and Yingxuan Zhuang*

Abstract

In robot handling, due to uneven load distribution, dynamic inertia force change and external interference, the angular acceleration of each joint easily exceeds the limit value, which seriously affects the handling trajectory accuracy. To realise coordinated and balanced motion of each joint of its dual arm, it is necessary to implement accurate control of each joint under the constraint of the coordinated trajectory of each joint. Therefore, an adaptive coordinated and balanced control method for a dual-arm robot considering the constraint of the joint angular acceleration is studied. Combined with Lagrange's second type equation, the dynamic model of the dual-arm robot is constructed, and the angular acceleration constraints of the robot's dual arm joints are set according to the model. With this constraint as input, an adaptive fuzzy model of the robot is constructed. Combining the sliding mode variable structure controller and adaptive fuzzy compensation controller, the robot adaptive fuzzy model is controlled by considering the expected trajectory of each joint of the robot arm as the input. In this study, sinusoidal vibration interference was employed with an amplitude of 0.5 m/s^2 and a frequency of 5 Hz. These parameters simulate common equipment vibration disturbances encountered in industrial environments. Results indicate that under normal operating conditions, the angular acceleration of each joint in the robot's manipulation process remained within limit values when controlled by this method, with no significant difference in angular acceleration between the two arms' joints. The actual motion trajectories of each joint in the dual-arm system closely matched the predefined target trajectories, with an average trajectory tracking error of 0.062 rad and a maximum deviation below 0.08 rad, enabling coordinated and balanced dual-arm operations. Under vibration disturbance conditions, this method still ensured that both arms' joints satisfied angular acceleration constraints during robotic manipulation while maintaining consistency between them. The average trajectory tracking error across all joints of the robotic arms was 0.078 rad, with a maximum deviation of 0.13 rad between the actual and desired trajectories. The overall control performance demonstrated stability, effectively countering external vibration disturbances to ensure coordinated, balanced, and stable operation of both arms during robotic tasks.

Key Words

Joint angular acceleration, dual-arm robot, adaptive, cooperative equilibrium control, sliding mode variable structure, fuzzy compensated control

1. Introduction

As an important force to promote contemporary industrial automation, intelligent manufacturing and medical rehabilitation, robot technology is gradually attracting widespread attention [1]. The dual-arm robot is equipped with two mechanical arms protruding from both sides of the robot body. The main types include dual-arm electronic measuring instruments, dual-arm cooperative robots, and dual-arm dual-power robots [2]. Its main features are strong adaptability, high flexibility and good coordination between the two arms. Dual-arm robot system is an extension of single-arm robot system. As an important branch of robot technology, it has been widely used in various fields with its unique dual-arm structure and highly flexible cooperative working ability. Compared with the one-arm robot, the two-arm robot needs to have stronger operation ability and adaptability, and to realise the coordinated and balanced operation of the two arms, which puts forward higher requirements for the control of the two-arm robot [3]. However, in actual working conditions, due to the irregular shape and different mass distribution of the transported object, the load will be unevenly distributed on both arms, which will cause uneven stress on each joint, and

* Chaozhou Power Supply Bureau Guangdong Power Grid Co., Ltd. Chaozhou, Guangdong 521000, China; e-mail: gu58064703@163.com; wotong859341@163.com; yuanzhang-tanv@163.com; zhicheng621928@163.com; yunzhuangtu-ija@163.com
Corresponding author: Yutong Zeng

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then affect the angular acceleration of the joint [4]. At the same time, the frequent changes in the speed and direction of the robot during handling will produce dynamic inertial force, and its magnitude and direction will continue to change, further increasing the fluctuation of joint angular acceleration. Under the combined action of these factors, the angular acceleration of each joint easily exceeds the limit value, which will not only reduce the running stability of the robot but also seriously affect the accuracy of the conveying trajectory and lead to the position deviation of the conveyed items. Therefore, it has become an urgent problem to ensure the coordination and balance of the two arms in the process of cooperative operation in order to achieve stable and efficient operation [5]. When the joint angular acceleration exceeds the limit value, it will cause differences in the response speed of each joint movement of the two arms. For example, the joint movement of the arm with a heavier load lags behind that of the other side, making it impossible for the two arms to maintain synchronised movement rhythm when performing the same handling task. This asynchronous movement will further disrupt the force balance and position coordination between the arms, ultimately resulting in the loss of the ability to coordinate and balance between the arms, affecting the stable execution of the handling task.

At present, some foreign scholars have carried out relevant research on the trajectory tracking control of robots, such as the trajectory tracking control method of a cable-driven robot proposed by Khalilpour *et al.* [6], which realises the trajectory tracking control of the robot by creating a dynamic model of the cable-driven robot, monitoring the robot's posture, and position parameters in real time with sensors, combining the dynamic model with the collected parameter data, writing control algorithm code, and deploying this code to the robot's proportional-integral-derivative (PID) control system. By combining dynamic modelling and sensor technology, this method can accurately track and control the trajectory of a robot during normal motion. However, for some robots in a special motion state, the tracking trajectory of this method will have some deviation owing to external interference, resulting in a poor final control effect. Selim and Alci [7] studied a control method for planar biped robots. This method obtains the phase information of the robot through the robot tilt angle, acceleration, ground reaction force, and other data collected by sensors. PID is used to control the stability of the supporting leg of the robot in the supporting state by using such phase information. Combining the Bezier curve algorithm, the foot trajectory of the robot under swing dynamics is obtained, and the stability control of the swing position of the robot leg is realised. The walking speed of the robot is controlled by adjusting the duration of the support and swing states in the gait cycle, as well as the swing speed and amplitude of the swing leg. By combining the PID and Bezier curve algorithms, this method can ensure that the planar biped robot can maintain good stability when walking at different speeds and terrain conditions. However, when an external vibration disturbance is applied to the robot, an unstable state easily appears in the robot walking under the control of this method, and the control effect is not ideal. The robot stability control method studied by Anastasiia *et al.* [8] constructs the robot kinematic model, designs the generalised scattering matrix control algorithm, uses sensors and other real-time sensors to obtain the state information in the process of robot motion, according to these information, make real-time adjustments to the control algorithm, and use the adjusted control algorithm to achieve robot stability control. The generalised scattering technology in this method can adapt to different types of robot systems, including non-linear, time-varying, and other complex systems, and has a strong adaptability. However, the derivation and application of the generalised scattering matrix usually depend on specific physical conditions and assumptions. For example, the scattering source is a localised scattering source, and the interaction with the scattered particles is limited in a limited space. These conditions and assumptions may limit the scope and accuracy of the generalised scattering matrix (GSM). In practical applications, if the conditions are not met or changed, it may be necessary to re-deduce or modify the GSM, which increases the difficulty and flexibility of applying of the algorithm. El Baji *et al.* [9] studied the tracking control method for the motion trajectory of the manipulator, which is based on the structure and dynamic characteristics of the serial manipulator and combined with new formulas to establish its motion equation. According to the motion equation, the motion parameters of the manipulator are obtained; The online control method is a control method that can collect real-time system motion parameters and adjust control strategies in real time based on parameter changes. Compared with real-time control method, it emphasises more on the explicit response and adjustment mechanism of control algorithms to parameter changes, rather than just focusing on real-time transmission of control signals. By combining this method with the obtained motion parameter data, real-time adjustment of the joint driving torque of the robotic arm is achieved to achieve trajectory tracking control of the robotic arm; Combining PID control with trajectory tracking to achieve motion control of the robotic arm. The introduction of the new formula of this method may make the realisation of the control algorithm more flexible and diverse and can be applied to different types of manipulators and control tasks. However, the explicit online control method needs to collect and process a large amount of data in real time and implement complex calculations and analyses, which may lead to an increase in the amount of calculation and a decrease in real time.

In the field of dual arm robot control research, sliding mode control has been applied to deal with system uncertainty due to its strong robustness. However, when used alone, it is prone to chattering and difficult to balance dual arm coordinated control under joint angular acceleration constraints. Although adaptive fuzzy control can approximate the non-linear characteristics of the system through fuzzy rules and achieve parameter adaptive adjustment, the control accuracy and response speed still need to be improved when dealing with dynamic coupling and external strong interference of dual arm robots.

Existing research mostly focuses on optimising or simply combining a single control method, lacking deep fusion design for collaborative equilibrium control of dual arm robots under joint angular acceleration constraints, and insufficient research on control stability under complex working conditions. Therefore, this study combines sliding mode variable structure control with adaptive fuzzy compensation control to construct a composite controller and introduces joint angular acceleration constraint analysis, filling the technical gap in existing research in this specific scenario and improving the collaborative control performance of dual arm robots in complex working conditions.

A sliding mode variable structure controller, also known as a sliding mode variable structure controller or sliding mode controller, is a special type of non-linear control strategy [10]. It is essentially a special class of non-linear control whose non-linearity is manifested in the discontinuity of the control. It is characterised by the control of the non-fixed structure of the system can be in a dynamic process according to the current state of the system purposefully changing, forcing the system in accordance with the predetermined "sliding mode" of the state trajectory movement [11]. The design principle of the controller is to design the switching hyperplane of the system according to the desired dynamic characteristics of the system, and the state of the system converges to the switching hyperplane from outside the hyperplane using a sliding mode controller. Once the system reaches the switching hyperplane, the control action will ensure that the system reaches the system origin along the switching hyperplane, and this process of sliding along the switching hyperplane to the origin is called sliding mode control [12]. Its main advantages include fast response speed, strong robustness, and simple physical realisation. It is widely used in the military, aerospace, robotics, and industrial fields [13]. An adaptive fuzzy compensation controller is an advanced control system that combines adaptive and fuzzy control techniques to form a control system with adaptive functions [14]. The controller does not require the control object to have an accurate mathematical model, but through the introduction of an adaptive law to learn the dynamic characteristics of the controlled object in real time, and according to the real-time changes in these characteristics, the parameters of the fuzzy controller are automatically updated and modified [15]. Its working principle is that in the control process, the controller uses fuzzy logic to defuzzify the input signals and generate the control outputs according to the fuzzy rules and affiliation function. Simultaneously, the adaptive law continuously adjusts the parameters of the fuzzy controller according to the real-time operation status of the system and error information, to realise precise control of the controlled object [16]. The main features of this controller are easy realisation, high control accuracy, strong adaptive ability, and high robustness. It is mainly used in electric power systems, robot control, aerospace, and industrial automation.

Based on the above analysis, this study combines sliding mode variable structure control with adaptive fuzzy compensation control to design an adaptive collaborative equilibrium control method for dual-arm robots considering joint angular acceleration constraints. This method can achieve adaptive collaborative balance control of the dual arm joints during robot motion, ensuring high collaborative balance of the robot's dual arm joints under angular acceleration constraints, thereby achieving efficient and stable collaborative task execution.

Compared to existing dual-arm robot control research, this paper innovatively establishes a dynamic model for dual-arm robots based on Lagrange's second-class equations, with joint angular acceleration constraints as the core control premise. By precisely dividing motion phases and integrating angular velocity curves, it maximises joint angular motion ranges while satisfying angular acceleration limit constraints. Compared to methods disregarding this constraint, the proposed approach significantly enhances the coordination and stability of dual-arm motion. Furthermore, it innovatively integrates the constructed dynamic model with the angular acceleration constraint to establish an adaptive fuzzy model for the dual-arm robot. Leveraging the universal approximation capability of fuzzy systems, this model enables real-time learning of the robot's dynamic characteristics. Compared to traditional fuzzy models, this approach improves approximation accuracy for system uncertainties, reducing the deviation between model outputs and actual system dynamics. A composite controller was designed by integrating sliding mode variable structure control with adaptive fuzzy compensation control. Sliding mode variable structure control ensures rapid response and strong robustness, while adaptive fuzzy compensation control achieves convergence through parameter adaptation. Compared to traditional PID control, the dual-arm robot achieves a 12.4% improvement in handling accuracy, reaching 96.4% precision. Even under vibration disturbances of 0.5 m/s^2 amplitude and 5 Hz frequency, it maintains joint angular acceleration within safe limits. This significantly outperforms existing control methods—whether solely combining sliding mode and adaptive control or neural networks—under complex operational conditions.

The innovation of this paper is as follows.

(1) This study focuses on the constraint of joint angular acceleration, which opens up a new direction for cooperative control of dual-arm robots. Based on Lagrange's second equation, the dynamic model is constructed, the angular acceleration constraint is accurately analysed, and the motion stages are innovatively divided. While satisfying the constraints, the joint angular motion range is maximised, and the coordination and stability of dual-arm motion are significantly improved.

(2) In this study, the dynamic model and angular acceleration constraints are combined to innovatively construct an adaptive fuzzy model. In view of the uncertainty of fuzzy system, the output function is redefined, the input vector is set accurately and the number of fuzzy rules is optimised by using its universal approximation characteristics. The model can learn and adapt to the dynamic characteristics

of robot system in real time and accurately, which lays a solid foundation for realising high-precision collaborative control and realises key innovation in fuzzy model construction.

(3) In this study, sliding mode variable structure control and adaptive fuzzy compensation control are organically combined for the first time to design a compound controller. Although sliding mode variable structure control has fast response and strong robustness, its effect is limited when facing the modelling error and external interference caused by the complex structure of dual-arm robot. With the help of fuzzy logic and adaptive law, adaptive fuzzy compensation control can effectively approach the uncertainty of the system and realise the adaptive compensation control for the cooperative balance of two arms. The combination of the two ensures that the dual-arm robot meets the constraint of joint angular acceleration in complex environment, realises high-precision trajectory tracking and significantly improves the cooperative control performance.

2. Adaptive Cooperative Equalisation Control Methods for Dual-Arm Robots

2.1 Dynamics Modelling and Joint Angular Acceleration Constraint Analysis of a Dual-Arm Robot

Combined with Lagrange's second equation, a model for the dynamics of the dual-arm robot is constructed, which can be written as follows:

$$\boldsymbol{\alpha}_A + \boldsymbol{C}^{\mathrm{T}} \boldsymbol{D} = \boldsymbol{A}(\boldsymbol{p})\, \ddot{\boldsymbol{p}} + \boldsymbol{B}(\boldsymbol{p}, \dot{\boldsymbol{p}})\, \dot{\boldsymbol{p}} \tag{1}$$

In the formula, $\boldsymbol{\alpha}_A$ represents the column vector of the dual-arm robot carrier and the driving force of each joint; $\boldsymbol{C} \in \mathbf{R}^{6\times 9}$ represents the kinematic Jacobian matrix corresponding to the end effectors of the left and right mechanical arms; The force and moment components of $\boldsymbol{D}$ acting on the gripper at the end of the left and right manipulator; $\boldsymbol{A}(\boldsymbol{p}) \in \mathbf{R}^{9\times 9}$ represents the symmetric positive definite inertia matrix of the dual-arm robot, where $\boldsymbol{p}$ is a generalised coordinate, indicating that the inertia matrix is a function of the generalised coordinate $\boldsymbol{p}$; $\ddot{\boldsymbol{p}}$ represents the second derivative of generalised coordinate $\boldsymbol{p}$ with respect to time, that is, generalised acceleration; $\boldsymbol{B}(\boldsymbol{p}, \dot{\boldsymbol{p}})\, \dot{\boldsymbol{p}} \in \mathrm{R}^{9\times 1}$column vector representing Coriolis force and centrifugal force; $\dot{\boldsymbol{p}}$ representing the first derivative of generalised coordinate $\boldsymbol{p}$ with time. Where $\boldsymbol{p}$ can be expressed as follows:

$$\boldsymbol{p} = \left[\boldsymbol{x}_0 \ \boldsymbol{Y}_0 \ \boldsymbol{\phi}_0 \ \boldsymbol{\phi}_{\text{left}}^T \ \boldsymbol{\phi}_{\text{right}}^T \right]^T \tag{2}$$

In the formula, $\boldsymbol{\phi}_{\text{left}}$ and $\boldsymbol{\phi}_{\text{lright}}$ denote the angle of rotation of each joint of the left and right arms of the robot; $\boldsymbol{\phi}_0$ denotes the robot carrier attitude angle; $\boldsymbol{x}_0$, $\boldsymbol{y}_0$ denote the coordinates of the center of mass position of the robot carrier.

The joint constraints of a dual-arm robot mainly include the joint angle, angular velocity, and angular acceleration constraints [17], which can be expressed by the inequality:

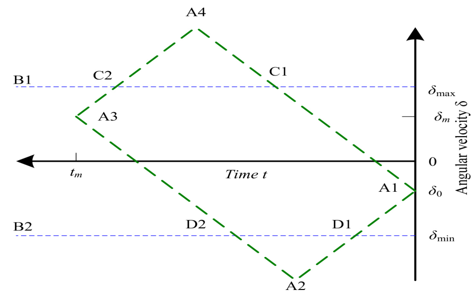

Figure 1. Joint angle constraint of dual-arm robot.

$$\begin{cases} \phi_{\min} \le \phi(t) \le \phi_{\max} \\ \left|\dot{\phi}(t)\right| \le \delta_{\max} \\ \left|\ddot{\phi}(t)\right| \le \varphi_{\max} \end{cases} \tag{3}$$

In the formula, $\phi(t)$, $\dot{\phi}(t)$, $\ddot{\phi}(t)$, respectively, are the joint angle, angular velocity, and angular acceleration constraints for a dual-arm robot at t moment; $\phi_{\max}$ and $\phi_{\min}$ are the maximum and minimum values of the robot's joint angles. $\delta_{\max}$ is the extreme value of the angular velocity of the robot joint; $\varphi_{\max}$ is the extreme value of the joint angular acceleration of the robot. The joint angle constraint problem for adaptive cooperative equalisation control of a dual-arm robot is presented in Fig. 1.

In the figure, the initial angular velocity of the dual-arm robot is δ_0, corresponding to point A1 the angular velocity of its target time t_m is δ_m, corresponding to point A3. Since the maximum joint angular acceleration of the dual arm robot during movement is $\varphi_{\max}$, then a parallelogram A1, A2, A3, A4 can be constructed. Points A1, A2, A3, and A4 represents the characteristic state points of a single typical joint of a dual arm robot during motion: A1 is the state point at the initial moment when the joint angular velocity is δ_0; A2 is the state point when the joint accelerates from the maximum angular acceleration $\varphi_{\max}$ to the maximum angular velocity δ_m; A3 is the state point at the target moment when the joint angular velocity is δ_m; A4 is the transition point during the process of joint deceleration from $\varphi_{\max}$ to δ_m. In this study, it is assumed that the motion characteristics of the joints corresponding to both arms are consistent. Therefore, the motion analysis of a single typical joint is used to represent the motion law of the joints corresponding to both arms. Not all joints reach the same target angular velocity δ_m, but the joints corresponding to both arms reach their respective target angular velocities δ_m at the same time. The slope of the four sides of the quadrilateral is $\varphi_{\max}$. According to the angular acceleration constraint, it can be determined that the angular velocity curve should be within the quadrangle. The angular velocity constraint of the robot can be indicated by two horizontal lines B1 : $\delta = \delta_{\max}$ and B2 : $\delta = \delta_{\min}$, where, two horizontal lines and parallelogram A1, A2, A3, A4 can intersect or not. Assuming that B1 intersects the quadrilateral at points C1 and C2, and B2 intersects the quadrilateral at points D1 and D2, the angular velocity curve of the robot should be within polygon A1, D1, D2, A3. C2, C1 to ensure that the robot will not exceed the angular velocity limit at any time. The shape of the appropriate joint angular velocity curve can be ensured for the angular acceleration and angular constraint of the robot joints at any time [18]. Choosing the appropriate type of joint angular velocity curve is crucial for solving constraint problems. The angular velocity curve that can best give play to the motion performance of the robot joint should be the curve that can move along the polygon A1, D1, D2, A3, C2, C1 to maximise the motion range of the robot joint angle (i.e., the integration of the angular velocity curve). Suppose that the motion process of the dual-arm robot is divided into three stages, namely: The joint angular acceleration at time 0-t_1 is the uniform acceleration motion of φ; A constant speed of motion at time t_1-t_2; The acceleration at time t_2 - t_m is the uniform decelerating motion of -φ; These three phases of motion can also be "uniform deceleration–uniform velocity– uniform acceleration", depending on the positive or negative of the robot joint angular acceleration φ.

It is assumed that the initial moment joint angle of the dual-arm robot is ϕ_0, then the 3-phase equations of motion for its motion process can be established as follows.

(1) When $0 \le t \le t_1$, the equation of motion of the dual-arm robot is given by

$$\begin{cases} \ddot{\phi}(t) = \varphi \\ \dot{\phi}(t) = \varphi t + \delta_0 \\ \phi(t) = \left(\frac{\varphi t^2}{2}\right) + \delta_0 t + \phi_0 \end{cases} \tag{4}$$

(2) When $t_1 \le t \le t_2$, the equation of motion of the dual-arm robot is given by

$$\begin{cases} \ddot{\phi}(t) = 0 \\ \dot{\phi}(t) = \varphi t_1 + \delta_0 \\ \phi(t) = \phi_0 - \left(\frac{\varphi {t_1}^2}{2}\right) + \delta_0 t + \varphi t_1 t \end{cases} \tag{5}$$

(3) When $t_2 \le t \le t_m$, the equation of motion of the dual-arm robot is given by

$$\begin{cases} \ddot{\phi}(t) = -\varphi \\ \dot{\phi}(t) = \varphi t_1 + \delta_0 - \varphi t + \varphi t_2 \\ \phi(t) = \phi_0 - \left(\frac{\varphi\left({t_1}^2 + {t_2}^2 + t^2\right)}{2}\right) + \delta_0 t + \varphi t_1 t + \varphi t_2 t \end{cases} \tag{6}$$

Then the joint angle ϕ_m of the dual-arm robot at the end point target time t_m can be expressed as:

$$\begin{aligned} \phi_m = \phi_0 - \left(\frac{\varphi\left({t_1}^2 + {t_2}^2 + {t_m}^2\right)}{2}\right) + \delta_0 t_m \\ + \varphi t_1 t_m + \varphi t_2 t_m \end{aligned} \tag{7}$$

The joint angular velocity of the dual-arm robot δ_m at the target moment t_m can be expressed as follows [19]:

$$\delta_m = \varphi t_1 - \varphi t_m + \varphi t_2 + \delta_0 \tag{8}$$

A system of equations is formed from (7) and (8), where the joint angular acceleration of the dual-arm robot φ and different moments t_1, t_2 can all be used as variables. One of the variables can be predetermined to obtain a unique solution to this system of equations. Two approaches are taken: one is to set the angular acceleration of the robot's joints φ as a fixed value, and at different moments t_1, t_2 as a variable; another is to make the moment $t_1 = t_2$, the angular acceleration of the robot's joints φ as a variable, in which case the joint angular acceleration obtained by solving for the φ is the minimum acceleration value to meet the requirement of cooperative and stable motion of the dual-arm robot. Considering the dual-arm robot joint angular acceleration constraints, a smaller motion joint angular acceleration is more likely to exceed the set constraints, and a larger joint angular acceleration is not conducive to the cooperative smoothness of the dual-arm robot motion and equalisation control accuracy [20]. Therefore, the ideal adaptive cooperative equilibrium control method for dual-arm robots should attempt to use smaller values of joint angular acceleration under the premise of satisfying the constraints of joint angular acceleration.

2.2 Adaptive Fuzzy Modelling of a Dual-Arm Robot Considering Joint Angular Acceleration Constraints

In order to realise the efficient and stable operation of dual-arm robot in collaborative tasks, it is necessary to accurately control the motion of each joint. Considering the constraint of joint angular acceleration, the reasonable motion trajectory can be planned to ensure the accuracy of robot operation under the condition of satisfying the extreme limit of joint angular acceleration. For example, in the handling task, the angular acceleration of each joint can be ensured within a safe range, and the coordinated and stable handling of both arms can be realised, thus improving the work quality and efficiency. Based on the above constructed dynamic model of the dual-arm robot and its joint angular acceleration constraint analysis, the fuzzy system output function of the dual-arm robot is defined in the following form [21]:

$$\widehat{Y}(\widehat{x}) = \zeta(\widehat{x})^T \Theta \tag{9}$$

In the formula, $\zeta(\widehat{x})$ denotes the fuzzy system regression vector for the dual-arm robot; $\widehat{x}$ and $\widehat{Y}(\widehat{x})$ denote the fuzzy system input and output vectors of the dual-arm robot, respectively; k denotes the number of affiliation functions in the fuzzy system; N denotes the total number of fuzzy rules for this fuzzy system; $\Theta = \left(\varphi^k, \cdots, \varphi^N\right)^T$ denotes the fuzzy system parameter vector of the dual-arm robot; Among them, N is the total number of fuzzy rules, k is the number of membership functions, and the parameter vector Θ covers the parameters from the k fuzzy rule to the N fuzzy rule, with a dimension of 33. The relationship between the setting of this dimension and the total number of fuzzy rules and membership functions is that by selecting parameters starting from the k rule, the parameters corresponding to the first $k-1$ redundant rules can be eliminated, which reduces the computational complexity and improves the real-time performance of the model while ensuring the approximation accuracy of the fuzzy system. $\zeta_k(\widehat{x})$ denotes the fuzzy system basis function of the dual-arm robot. $\zeta_k(\widehat{x})$ can be expressed as follows:

$$\zeta_k(\widehat{x}) = \frac{\prod_{j=1}^{m} \eta_{\chi_j}^{k}(\widehat{x}_j)}{\sum_{j=1}^{N}\left(\prod_{j=1}^{m} \eta_{\chi_j}^{k}(\widehat{x}_j)\right)} \tag{10}$$

In the formula, $\eta_{\chi_j}^{k}(\widehat{x}_j)$ denotes the Gaussian affiliation function of this fuzzy system, where, $j = 1, \cdots, N$. m denotes the total number of joints in the dual arm robot. In this study, the Motoman SDA10 dual arm robot has seven joints in a single arm, so the total number of joints in both arms is m=14. The reason for choosing this value is that the input vector of the fuzzy system needs to cover the state information of all joints in both arms, in order to achieve accurate approximation of the dynamic characteristics of the entire dual arm robot system and ensure that the adaptive fuzzy model can comprehensively reflect the impact of joint motion on the overall performance of the system.

Considering the existence of uncertainty in the fuzzy system of the dual-arm robot, that is, there are errors or that cannot be solved for the matrix in (1) of the dual-arm robot dynamic models $A(p)$ and $B(p, \dot{p})$, the adaptive fuzzy model of this fuzzy system can be established using the universal approximation property of this fuzzy system of the robot.

Set $k = 3$, then the Gaussian affiliation function of the fuzzy system of the dual-arm robot $\eta_{\chi_j}^{3}(\widehat{x}_j)$ is:

$$\eta_{\chi_j}^{3}(\widehat{x}_j) = \exp\left[-\frac{1}{2}\left(\frac{\widehat{x}_j - 1.24}{0.7}\right)^2\right] \tag{11}$$

Due to the matrix $A(p)$ is only the function with respect to variables p, define the input vector of the fuzzy system of the dual-arm robot as $\widehat{x} = (\varphi_0, \varphi_1, \varphi_2, \cdots, \varphi_m)$, of which, φ_0 denotes the initial joint angular acceleration of the dual-arm robot; $\varphi_1 \sim \varphi_m$ represents the joint angular acceleration of t_1 moment to target moment t_m. The total number of fuzzy rules for this fuzzy system is set to $N = 3^5$, and the constructed fuzzy system model of the dual-arm robot is:

$$\widehat{A}(\widehat{x}, p_a) = \begin{bmatrix} \widehat{a}_{11}(\widehat{x}, p_{a_{11}}) \cdots \widehat{a}_{1m}(\widehat{x}, p_{a_{1m}}) \\ \vdots \quad \vdots \quad \vdots \\ \widehat{a}_{m1}(\widehat{x}, p_{a_{m1}}) \cdots \widehat{a}_{\mathrm{mm}}(\widehat{x}, p_{a_{\mathrm{mm}}}) \end{bmatrix} \tag{12}$$

Among them, $\widehat{a}_{\mathrm{ji}}(\widehat{x}, p_{a_{\mathrm{ji}}})$ can be expressed as follows:

$$\widehat{a}_{\mathrm{ji}}(\widehat{x}, p_{a_{\mathrm{ji}}}) = \zeta_{a_{\mathrm{ji}}}^{T}(\widehat{x})\,\varphi_{a_{\mathrm{ji}}} \tag{13}$$

In the formula, $\varphi_{a_{\mathrm{ji}}}$ denotes the vector of adaptive adjustment parameters for the model. The total number of fuzzy rules of this fuzzy system is set as $N = 3^{10}$. The fuzzy system model of the dual-arm robot constructed is as follows:

$$\widehat{B}(\widehat{x}, p_b) = \begin{bmatrix} \widehat{b}_{11}(\widehat{x}, p_{b_{11}}) \cdots \widehat{b}_{1m}(\widehat{x}, p_{b_{1m}}) \\ \vdots \quad \vdots \quad \vdots \\ \widehat{b}_{m1}(\widehat{x}, p_{b_{m1}}) \cdots \widehat{b}_{\mathrm{mm}}(\widehat{x}, p_{b_{\mathrm{mm}}}) \end{bmatrix} \tag{14}$$

In the formula, $p_{b_{\mathrm{ji}}}$ represents the j parameter in the rule i, and $\widehat{b}_{\mathrm{ij}}(\widehat{x}, p_{b_{\mathrm{ij}}})$ represents the output part of the fuzzy rule calculated according to the state vector $\widehat{x}$ and the parameter $p_{b_{\mathrm{ij}}}$, which is an element of the fuzzy system output matrix $\widehat{B}$.

Among them, $\widehat{b}_{\mathrm{ji}}(\widehat{x}, p_{b_{\mathrm{ji}}})$ can be expressed as follows:

$$\widehat{b}_{\mathrm{ji}}(\widehat{x}, p_{b_{\mathrm{ji}}}) = \zeta_{b_{\mathrm{ji}}}^{T}(\widehat{x})\,\varphi_{b_{\mathrm{ji}}} \tag{15}$$

In the formula, $\varphi_{b_{\mathrm{ji}}}$ represents the adaptive adjustment parameter vector of the model. Based on this, the final adaptive fuzzy model of the constructed dual-arm robot is as follows:

$$\boldsymbol{F} = \widehat{\boldsymbol{A}}(\widehat{x}, p_a)\,\ddot{\boldsymbol{p}} + \widehat{\boldsymbol{B}}(\widehat{x}, p_b)\,\dot{\boldsymbol{p}} \tag{16}$$

Joint angle constraint is ensured through the following mechanism: Joint velocity constraint ensures that the joint angular velocity is always within the range of [$\delta_{\min}$, $\delta_{\max}$] through motion phase division, and joint angular velocity is the first derivative of the joint angle with time. By integrating the angular velocity curve in the time domain, the range of joint angle changes can be obtained. Combined with the initial joint angle φ_o, it can ensure that the joint angle is always within the range of [$\varphi_{\min}$, $\varphi_{\max}$]. The adaptive fuzzy model incorporates the joint angular acceleration constraint $\varphi_{\max}$ during the model construction process. By adjusting the model parameters in real-time through adaptive laws, the joint angular acceleration output by the model always does not exceed $\varphi_{\max}$. Furthermore, through the correlation between angular velocity and angular acceleration, the joint angular constraint is indirectly ensured to be satisfied.

3. Adaptive Cooperative Equalisation Control for Dual-Arm Robots

When the constructed adaptive fuzzy model of the dual-arm robot is precisely known and there is no external interference, the adaptive cooperative equilibrium control of the dual-arm robot can be realised directly by designing a sliding mode controller for the model. However, in reality, the complex structure of the dual-arm robot system is prone to modelling errors and external uncertainties [22], and it is difficult to achieve adaptive cooperative equilibrium control of the dual-arm robot by relying solely on the sliding mode controller. For this reason, in this study, based on the sliding mode variable structure controller, an additional adaptive fuzzy compensation controller is used to achieve adaptive control of the robot's dual-arm cooperative balance. The overall control structure is shown in Fig. 2.

Taking the desired trajectories of each joint in the motion of the dual-arm robot as input, the adaptive

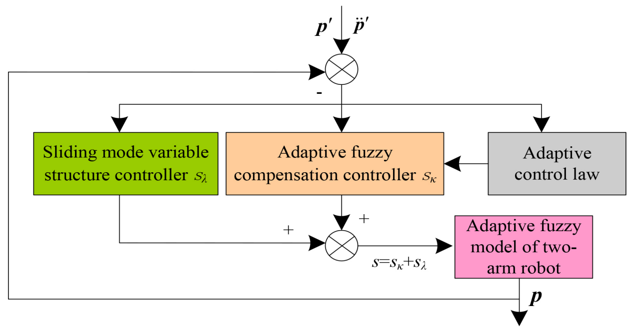

Figure 2. Overall structure of adaptive collaborative equalisation control for dual-arm robot.

cooperative equilibrium control of the dual-arm robot in various complex environments is realised based on the adaptive fuzzy model of the dual-arm robot constructed by considering the angular velocity constraints of the joints in the set composite control law through the composite control mode combining the sliding-mode variable-structure control and adaptive fuzzy compensatory control. The output of the actual motion trajectory of each joint of the dual-arm robot is output after the control. The realisation process of the two control modules is as follows.

(1) *Sliding Mode Variable Structure Control:*

Defining the angular acceleration trajectory tracking error $\varepsilon(t)$ of the joint motion of a dual-arm robot and the sliding mode tangent function $\gamma(t)$ as:

$$\begin{cases} \varepsilon(t) = p(t) - p'(t) \\ \gamma(t) = \dot{\varepsilon}(t) + \beta\varepsilon(t) \end{cases} \tag{17}$$

In the formula, $p(t)$ denotes the actual trajectories of the joints of the dual-arm robot at the moment t; $p'(t)$ denotes the desired trajectory of the joints of the dual-arm robot at that moment; $\beta = diag(\beta_1, \beta_2, \cdots, \beta_m)$ denotes the matrix of constant coefficients; $\dot{\varepsilon}(t)$ denotes the tracking error of the joint motion trajectory after control.

The core theoretical basis of control law design based on the system dynamic model (1) is integrated throughout the entire control design process. To reduce the complexity of control design, a nominal model is constructed based on (1). Due to the fact that in the adaptive fuzzy model (16), $\widehat{A}(\widehat{x}, p_a) \approx A(p)$, $\widehat{B}(\widehat{x}, p_b) \approx B(p)$ are substituted into (1), and the output of the sliding mode controller is $s_\lambda = \alpha_A + CD$, the nominal model can be obtained:

$$s_\lambda = \widehat{\boldsymbol{A}}\ddot{\boldsymbol{p}} + \widehat{\boldsymbol{B}}\dot{\boldsymbol{p}} \tag{18}$$

In the formula, s_λ denotes the sliding mode variable structure controller. This step preserves the core dynamic characteristics of (1) through fuzzy approximation, transforming complex non-linear dynamic models into nominal models that can be directly used for control design, providing a foundation for subsequent controller derivation.

The guarantee mechanism and mathematical proof for the convergence of adaptive law driven parameter estimation values to true values are as follows.

(1) *Definition of Parameter Estimation Error and Adaptive Law:*

Define the parameter estimation error: Let ϕ^* be the true values of the parameters for the inertia matrix approximation term $\widehat{A}(\widehat{x}, p_a)$ and the Coriolis/centrifugal force matrix approximation term $\widehat{B}(\widehat{x}, p_b)$ in the fuzzy model, and $\widehat{\phi}$ be the parameter estimation values. Then the parameter estimation error is $\widetilde{\phi} = \phi^* - \widehat{\phi}$. The adaptive law is designed as $\dot{\widehat{\phi}} = \Gamma\zeta(x)\gamma$, where Γ is a positive definite gain matrix ($\Gamma = \Gamma^T > 0$), $\zeta(x)$ is the fuzzy system basis function (derived from the Gaussian membership function in 10), and γ is the sliding mode switching function (17).

(2) *Construction of Extended Lyapunov Function:*

Construct the extended Lyapunov function: Select $V = 0.5\gamma^T A\gamma + 0.5\widetilde{\phi}^T\Gamma^{-1}\widetilde{\phi}$, where A is the symmetric positive definite inertia matrix of the system (1). Since $A > 0$ and $\Gamma > 0$, $A > 0$ holds for all$(\gamma, \widetilde{\phi}) \neq (0, 0)$, and $V = 0$ if and only if $\gamma = 0$ and $\widetilde{\phi} = 0$, which satisfies the positive definiteness requirement of the Lyapunov function.

(3) *Calculation and Analysis of the Time Derivative of V:*

Taking the time derivative of V gives $\dot{V} = 0.5\dot{\gamma}^T A\gamma + 0.5\gamma^T\dot{A}\gamma + 0.5\gamma^T A\dot{\gamma} + \widetilde{\phi}^T\Gamma^{-1}\dot{\widetilde{\phi}}$. Substitute $\dot{\gamma} = A^{-1}(\theta - B\gamma - A\beta\gamma - \mu\mathrm{sgn}\gamma)$ (derived from the system model in (1) and the sliding mode controller in (20)) and $\dot{\widehat{\phi}} = \Gamma\zeta(x)\gamma$ into the above equation. Meanwhile, using the universal approximation property of the fuzzy system, let $\theta = \zeta(x)\phi^* + \sigma$. After rearrangement, obtain $\dot{V} = \gamma^T(\theta - B\gamma - A\beta\gamma - \mu\mathrm{sgn}\gamma) + 0.5\gamma^T\dot{A}\gamma + \widetilde{\phi}^T\zeta(x)\gamma$. Substitute $\widetilde{\phi} = \phi^* - \widehat{\phi}$and $\theta = \zeta(x)\phi^* + \sigma$ into $\widetilde{\phi}^T\zeta(x)\gamma$, resulting in $\widetilde{\phi}^T\zeta(x)\gamma = \gamma^T\zeta(x)\phi^* - \gamma^T\zeta(x)\widehat{\phi} = \gamma^T(\theta - \sigma) - \gamma^T\zeta(x)\widehat{\phi}$. Further substitute this into the expression of $\dot{V}$and simplify by combining with the adaptive law $\dot{\widehat{\phi}} = \Gamma\zeta(x)\gamma$, leading to: $\dot{V} \leq -\gamma^T(B + A\beta)\gamma - \mu\gamma^T\mathrm{sgn}\gamma + \gamma^T\sigma + 0.5\gamma^T\dot{A}\gamma$.

(4) *Proof of Convergence and Boundedness:*

Since the derivative $\dot{A}$ of the inertia matrix A satisfies $\gamma^T\dot{A}\gamma \leq k\gamma^T\gamma$. Then $\dot{V} \leq -\eta\gamma^T\gamma \leq 0$, according to Barbalat's lemma, as $t \to \infty$, $\gamma \to 0$ and $\gamma \in L_2 \cap L_\infty$. Since γ is bounded, $\dot{\widehat{\phi}} = \Gamma\zeta(x)\gamma$ is bounded, so $\widehat{\phi}$ is uniformly continuous; furthermore, because $\gamma \in L_2$, it can be concluded that $\widehat{\phi} \to \phi^*$, meaning the parameter estimation values converge to the true values. Meanwhile, $\widetilde{\phi} = \phi^* - \widehat{\phi}$. Since $\phi^*$and $\widehat{\phi}$ are both bounded, $\widetilde{\phi}$is bounded, thus realising the boundedness guarantee of the parameter estimation error.

According to (17) and (18), it can be deduced that:

$$\dot{\gamma} = \ddot{\varepsilon} + \beta\dot{\varepsilon} = \beta\dot{\varepsilon} + \widehat{\boldsymbol{A}}^{-1}\left[s_\lambda - \widehat{\boldsymbol{B}}\dot{\boldsymbol{p}}\right] - \ddot{\boldsymbol{p}}' \tag{19}$$

This leads to the design of a sliding mode variable structure controller s_λ for:

$$s_\lambda = \widehat{\boldsymbol{A}}\left(\ddot{\boldsymbol{p}}' - \beta\dot{\varepsilon} - \boldsymbol{\mu}\mathrm{sgn}\gamma\right) + \widehat{\boldsymbol{B}}\dot{\boldsymbol{p}} \tag{20}$$

In the formula, $\mu = [\mu_1, \mu_2, \cdots, \mu_m]^T$ denotes the matrix of convergent constant coefficients, $\ddot{\varepsilon}$ represents the acceleration of the error amount. At the same time, there is:

$$\mathrm{sgn}\gamma = \left[\mathrm{sgn}\left(\gamma_1\right), sgn\left(\gamma_2\right), \cdots, sgn\left(\gamma_m\right)\right]^T \tag{21}$$

And the conditional equation to be fulfilled for $\mathrm{sgn}\gamma_j$ is:

$$\mathrm{sgn}\gamma_j = \begin{cases} -1, \gamma_j < 0 \\ 1,\ \gamma_j > 0 \end{cases} \tag{22}$$

Due to the modelling error of dual-arm robot and the uncertainty caused by external disturbance, such that the term $\widehat{A}^{-1}\left(-p\ddot{p} - B\left(p, \dot{p}\right)\dot{p}\right) = \theta$, assuming that this uncertainty term does not exist in the modelling of the dual-arm robot, then $\theta = 0$, substituted into the dynamic model of the dual-arm robot, and the conclusion can be drawn:

$$\ddot{\varepsilon} + \beta\dot{\varepsilon} + \boldsymbol{\mu}\mathrm{sgn}\gamma = 0 \tag{23}$$

That is:

$$\dot{\gamma} = -\boldsymbol{\mu}\mathrm{sgn}\gamma \tag{24}$$

The uncertainty term θ is a concentrated manifestation of system modelling errors and external disturbances in control design, and its application and compensation run through the design of control laws. First, θ is obtained by subtracting the true inertia matrix, Coriolis force, and centrifugal force matrix of the system from the approximation matrix in the adaptive fuzzy model, and then multiplying them with the generalised acceleration and generalised velocity, respectively, to obtain the difference. This quantifies the uncertainty caused by the coupling of model approximation deviation and system state, providing a clear object for subsequent compensation design. In control law design, θ is substituted into the original dynamic model of the system to obtain a system model containing uncertainty. Based on this, the control law needs to simultaneously address the dynamic characteristics of the nominal model and the influence of θ. To counteract the interference of θ on control accuracy, an adaptive fuzzy compensation controller is designed, utilising the general approximation ability of fuzzy systems to non-linear functions, so that the output of the compensation controller can dynamically approximate the changes in θ. By combining the compensation controller with the sliding mode variable structure controller to form a total control law, and substituting the total control law into the system model containing θ, the output of the compensation controller can approximately cancel out the effect of θ, making the actual operating characteristics of the system approach the ideal characteristics based on the nominal model, thereby eliminating the uncertainty caused by modelling errors and external disturbances. Experimental verification shows that after compensation, the joint trajectory tracking error of the system is significantly reduced, proving the effectiveness of the θ compensation mechanism and ensuring that the robot can maintain stable coordinated and balanced motion under complex working conditions.

The derivation of the correlation between (20)–(24) needs to be based on a nominal model and is not directly related to the uncertainty term θ. The specific derivation process is as follows: starting from the convergence condition of the derivative of the sliding mode switching function, the sliding mode switching function is defined as the linear combination of the first-order derivative of the trajectory tracking error and the error itself. After taking the derivative, combined with the definitions of trajectory tracking error, error derivative, and error second-order derivative, the generalised acceleration of the system is represented by the expected trajectory acceleration, error related term, and convergence term. By substituting the generalised acceleration expression into the output formula of the sliding mode controller based on the nominal model, and combining it with the actual situation in the experiment where the expected trajectory is in a uniform speed stage and the expected trajectory acceleration and velocity are approximately zero through trajectory planning, (20) can be simplified. This clarifies the derivation logic of (20)–(24) and corrects potential logical confusion in the original expression. At the same time, the sliding surface serves as the core link between stability analysis and parameter update mechanism. In stability analysis, a Lyapunov function is constructed with the sliding surface as the core. The stability of the system is determined by analysing the sign of the derivative of the function. The convergence of the sliding surface directly determines the convergence of the trajectory tracking error, thereby ensuring system stability; In the parameter update mechanism, the parameter update law uses the sliding surface as the feedback signal. When the sliding surface is not zero, it indicates the existence of trajectory deviation or uncertainty in the system, triggering parameter update to adjust the output of the adaptive fuzzy compensation controller to approximate the uncertainty term until the sliding surface converges to zero, and the parameter update tends to be stable, ensuring that the mathematical derivation logic of the entire control design is coherent and traceable. Based on this, by obtaining the sliding mode tangent function, various interference uncertainties in the robot motion can be eliminated, and effective control of the trajectories of the arms in the motion of the dual-arm robot can be realised. Thus, the dual-arm robot achieves asymptotic stability.

(5) Adaptive Fuzzy Compensatory Control:

Continuing to utilise the universal approximation property of fuzzy control based on the sliding-mode variable structure control of a dual-arm robot with an uncertainty term θ, the approximation is implemented, and the adaptive fuzzy compensation controller is designed to realise the adaptive cooperative equalisation compensation control of the dual-arm robot. The approximation function of the adaptive fuzzy compensation control is set as follows:

$$\theta\left(\widehat{x}\right) = \sigma + \widehat{\theta}\left(\widehat{x}, \Theta\right) \tag{25}$$

In the formula, σ denotes the minimum approximation error. Where, $\widehat{\theta}\left(\widehat{x}, \Theta\right)$ can be expressed as follows:

$$\widehat{\theta}\left(\widehat{x}, \Theta\right) = \boldsymbol{\varphi}\zeta^T\left(\widehat{x}\right) \tag{26}$$

In the formula, φ denotes the vector of adaptive adjustment parameters, that is, the vector of angular acceleration of the joints of the dual-arm robot. Define the state vector of the fuzzy system of the dual-arm robot as $\widehat{x} = \left[\varepsilon^T, \dot{\varepsilon}^T\right]^T$. The error state equation of the fuzzy system of the dual-arm robot is:

$$\dot{\widehat{x}} = \widehat{\boldsymbol{A}}\widehat{x} + \widehat{\boldsymbol{B}}\xi \tag{27}$$

In the formula, $\xi = \mu \mathrm{sgn}\gamma + \zeta^T(\widehat{x}) + \sigma$. Based on this, the control law s_κ of this designed adaptive fuzzy compensation controller is:

$$s_\kappa = \zeta \boldsymbol{B}\boldsymbol{q}\widehat{x}\mathbf{R}^{-1}\zeta(\widehat{x})^T \widehat{\boldsymbol{A}}\widehat{\boldsymbol{B}}^{-1} \tag{28}$$

In the formula, q denotes the control law coefficients. Combined with the sliding mode variable structure controller equations of (20), the final control law of the composite controller can be derived as:

$$s = s_\kappa + s_\lambda \tag{29}$$

This study adopts Lyapunov stability theory as the criterion for controller convergence and stability. Define the Lyapunov function as:

$$\mathrm{V} = 0.5\gamma^T A\gamma \tag{30}$$

Calculate the time derivative of V:

$$\widetilde{V} = 0.5\gamma^T A\gamma + 0.5\gamma^T \tilde{A}\gamma + 0.5\gamma^T A\gamma \tag{31}$$

Derive $\widetilde{V} \leq -\eta\gamma^T\gamma$ (where η is a normal number). According to Lyapunov stability theory, when $\widetilde{V} \leq 0$ and only when $\gamma = 0$, $\widetilde{V} = 0$, it indicates that the sliding mode switching function γ asymptotically converges to zero, thereby proving that the dual arm robot system is asymptotically stable and the controller can achieve stable control of the system.

Under the composite control law, the left and right arms of the dual-arm robot can achieve coordinated equilibrium stabilised motion under the premise of satisfying the joint angular acceleration constraints during the motion process.

4. Analysis of Experimental Results

The Motoman-SDA10 dual arm robot selected in this study has a single arm consisting of seven joints (links), and the overall structure adopts a series configuration. The left and right arm structures are symmetrical, and each joint can achieve independent rotational motion. The joint rotation range is –170°∼+170°. Motoman-SDA10 dual-arm robot is selected as the experimental object, and the method proposed in this study is used for adaptive cooperative balance control of the robot, and the actual control effect of this research method is tested by the control results. In order to verify the effectiveness of the proposed adaptive cooperative equalisation control method on the dual-arm robot more comprehensively, a series of simulation experiments were carried out. The purpose of the simulation experiment is to simulate the operation of the dual-arm robot in the actual handling task, to evaluate the performance of the control method under different conditions in the simulation environment, and to accurately build the model of the Motoman-SDA10 dual-arm robot. According to the actual specifications of the robot, as shown in Table 1, the accurate setting of these parameters is very important to simulate the real dynamic characteristics of the robot and ensure the reliability of the simulation results. At the same time, considering the effect of gravity in the actual working scene, the acceleration of gravity is set to the standard value of 9.8m/s, and the friction between the robot and the working surface is simulated, and the appropriate friction coefficient is set according to the actual material characteristics, so as to more truly reflect the stress of the robot in the handling process.

Table 1
Key Parameters of Experimental Robot.

Argument	Detailed Information
Brand	Yaskawa
Load	10kg/Arm
Armspan	Horizontal arm span 1970 mm;Vertical extension length 1440 mm
Positioning accuracy	±0.1 mm
Number of axes	15 axis
Range of action	Axis of rotation –170° ∼+170°, Maximum speed of wrist rotation axis 400°/s
Use	Handling, assembly/subassembly and others
Control cabinet	DX100
Bulk quality	220 kg
Power supply capacity	2.5 kVA
The allowable torque of the wrist rotation axis	31.4N· m

In the experiment, the cooperative balance and accuracy of the robot's dual arm in the handling process are examined by designing an experimental robot cooperative handling task under the control of the proposed method. The experimental scene is shown in Fig. 3.

In the experiment, the angular acceleration constraints are set for the dual-arm joints during the handling process of the experimental robot, and the changes in the angular acceleration values of the dual-arm joints during the handling process of the experimental robot before the method proposed in this paper is controlled and the changes in the angular acceleration values of the dual-arm joints under the method proposed in this paper

Figure 3. Experimental scene diagram.

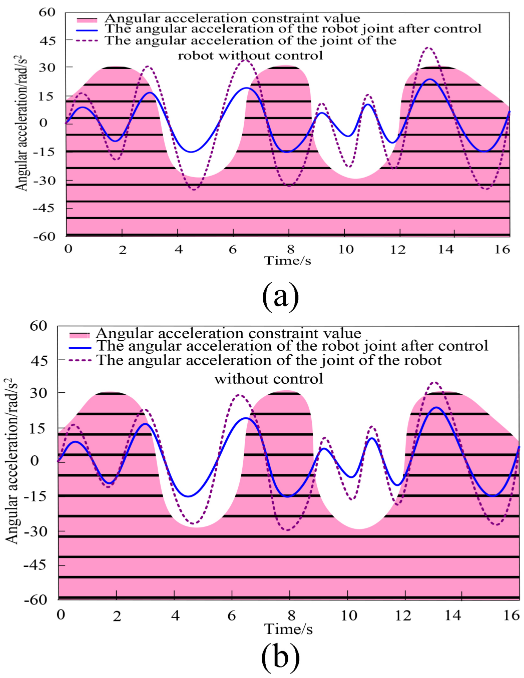

Figure 4. The method presented in this paper controls the angular acceleration changes of the lower arm joints of the front and rear experimental robots; (a) Angular acceleration of lower joint of left arm of experimental robot and (b) Angular acceleration of lower joint of right arm of experimental robot.

are statistically analysed. The comparative results of the left and right lower joints of the experimental robot, for example, are shown in Fig. 4.

As shown in Fig. 4, before using the control method in this study, the angular acceleration values of both arm joints during the handling process of the experimental robot exceeded the constraint values. The angular acceleration values of the left arm joints significantly exceeded the constraint values at 4.25 s, 6.21 s, 13.19 s, and the angular acceleration values of the right arm joints were significantly higher than the constraint values only at 13.19 s. It can be seen that, during the handling process of the experimental robot, the joint angular acceleration of both arms exceeds the limit, and the joint angular acceleration values of the left and right arms are different, so the coordination and balance of both arms are not ideal. Under the control of this method, the joint angular acceleration of the dual-arm of the experimental robot during the handling process can be maintained within the constraint value, and the joint angular acceleration of the left and right arms are consistent. It can be seen that under the control of this method, the angular acceleration of each joint of the experimental robot during the handling process can be guaranteed not to exceed the limit value, and the coordination balance of its dual-arm can be significantly improved to achieve the cooperative handling effect.

The desired trajectories are set for the joints of both arms of the experimental robot during the handling process, and the actual trajectories of the joints of the experimental robot during the cooperative handling process are obtained after the control of the method in this paper, and the two trajectories are shown in Fig. 5.

As shown in Fig. 5, under the control of the proposed method, the actual motion trajectories of each joint in the experimental robot's dual arms closely match the predefined target trajectories. Only minor trajectory deviations occur during the initial motion phase of each joint, with an average trajectory tracking error of 0.062 rad and a maximum deviation not exceeding 0.08 rad. It can be seen that the method in this study can realise the coordinated and balanced control of each joint of the dual-arm of the experimental robot during the handling process, and the control effect is remarkable.

To test the control effect of this method under a complex situation of external disturbance, vibration interference is applied to the base of the experimental robot. The results are shown in Fig. 6, which show that the angular acceleration of the lower joints of both arms of the experimental robot exceeds the limit during the handling process, and the angular acceleration of the lower joints of both arms of the experimental robot exceeds the limit after starting the control of the method.

It can be seen from the analysis of Fig. 6 that under vibration interference, the angular acceleration value of the lower joint of the dual-arm during the handling process of the experimental robot obviously exceeds the limit, and the exceeding value is higher than that without the interference. At the same time, the difference in the angular acceleration of the joint of the dual-arm also increases significantly, the motion coordination and balance of the robot's dual-arm are poor, and it is easy to lose stability. Under the control of the method in this study, the angular acceleration of the dual-arm joints during the handling of the experimental robot can still be guaranteed not to exceed the limit value, which can meet the requirements of the expected constraint angular acceleration limit value. Moreover, the angular acceleration of the dual arm is relatively consistent, and the dual arm still maintains good coordination and balance.

Under this vibration interference, the tracking errors of the dual-arm joints of the experimental robot controlled by the method of this paper are shown in Table 2.

It can be seen from Table 2 that under the vibration interference, the maximum value of the tracking error of each joint motion track of the experimental robot's dual-arm after the control of this method is 0.13 rad, where the average error of the upper and lower joints of the left

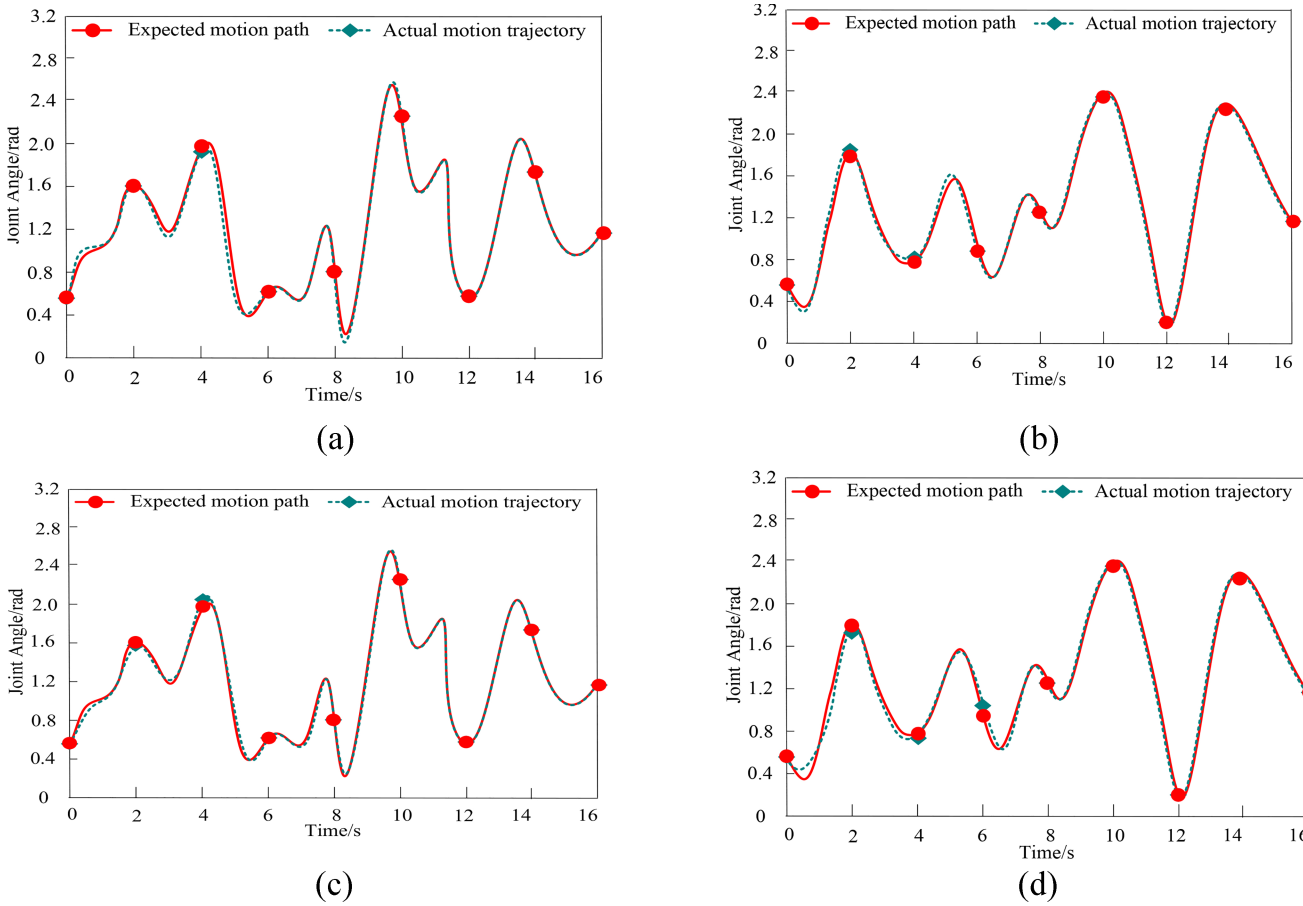

Figure 5. Expected and actual motion trajectories of each joint of both arms of experimental robot: (a) Motion trajectory of the upper joint of the left arm of the experimental robot; (b) Motion trajectory of the lower joint of the left arm of experimental robot; (c) Motion trajectory of the upper joint of the right arm of the experimental robot; and (d) Motion trajectory of lower right arm joint of experimental robot.

Table 2
Motion Trajectory Tracking Errors of Each Joint of Both Arms of the Experimental Robot Controlled by this Method Under Vibration Interference (rad).

Time/s	Upper Joint of Left Arm	Lower Joint of Left Arm	Upper Joint of Right Arm	Right Lower Arm Joint
2	0.11	0.11	0.12	0.11
4	0.09	0.11	0.10	0.12
6	0.13	0.08	0.09	0.09
8	0.08	0.10	0.10	0.10
10	0.10	0.07	0.08	0.08
12	0.09	0.08	0.06	0.07
14	0.06	0.07	0.07	0.05
16	0.04	0.05	0.05	0.06
18	0.02	0.04	0.02	0.03

arm is 0.080 rad and 0.079 rad, respectively, the average error of the upper and lower joints of the right arm is 0.077 rad and 0.079 rad, respectively, and the average value of the overall tracking error of each joint motion track of the dual-arm is 0.078 rad. It can be seen that in the presence of vibration interference, although the maximum value of the tracking error of each joint motion track of the experimental robot's dual-arm after the control of this method is slightly higher than that without interference, the overall tracking error value is still lower than 0.08 rad.

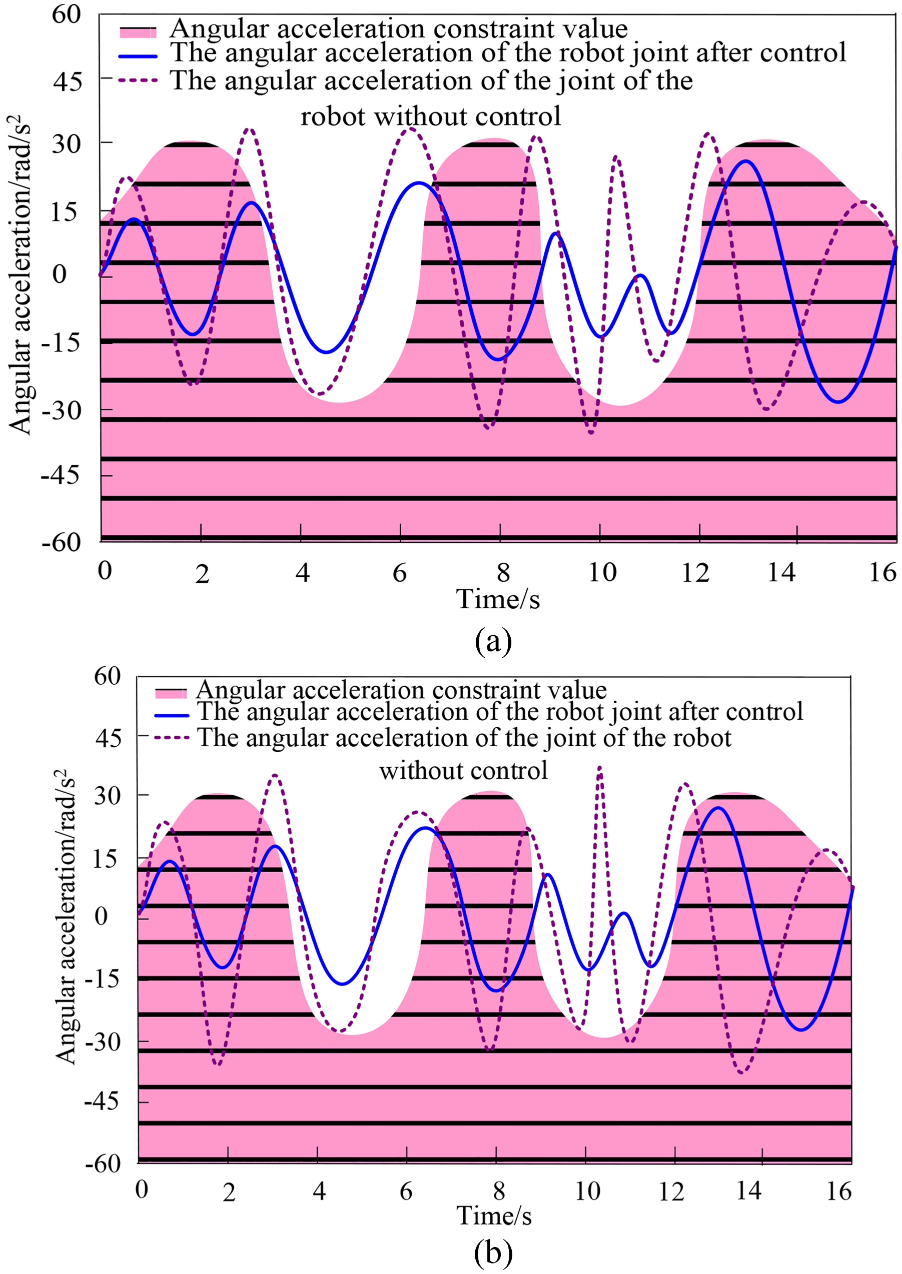

Figure 6. Under vibration interference, the angular acceleration of both arms joints of front and rear experimental robots is controlled by this method: (a) Angular acceleration of lower joint of left arm of experimental robot and (b) Angular acceleration of lower joint of right arm of experimental robot.

The overall control performance of this method is stable and can effectively deal with external vibration interference and ensure coordination balance and stability of the dual-arm during the handling of the experimental robot.

In order to evaluate the performance of the proposed control method in the handling operation of dual-arm robot more comprehensively, a comparative experiment was conducted with another common collaborative control method of dual-arm robot based on traditional PID control, focusing on the accuracy of handling operation, which was measured by three indicators: accuracy, recall and F1 score. The experimental results are shown in Table 3.

As can be seen from Table 3, the method proposed in this paper has obvious advantages compared with the traditional PID control method. In terms of accuracy, this method reaches 96.4%, which is 12.4% points higher than the traditional PID control method, which shows that this method can transport the parts to the target position more accurately and reduce the occurrence of wrong handling. In terms of recall rate, this method is 95.8%, while the traditional PID control method is only 84.6%, which means that this method can grab and carry all the parts that need to be transported more effectively, with fewer omissions. F1 score as a comprehensive measure, the 96.1% of this method is much higher than the 89.3% of the traditional PID control method, which further shows that the overall performance of this method is better in handling accuracy. This is mainly due to the

Table 3
Comparison Results of Accuracy of Different Methods.

Index	The Method in This Paper	Traditional PID Control Method
Accuracy rate	96.4%	84.0%
Recall rate	95.8%	85.2%
F1 score, f score, f measure	96.1%	84.6%

constraint of joint angular acceleration. By constructing an adaptive fuzzy model and adopting a compound control strategy, it can better cope with the complex situations such as uneven load distribution, dynamic inertia force changes and external interference, and adjust the robot's trajectory in real time, thus achieving more accurate handling operations. However, due to its own limitations, the traditional PID control method is difficult to deal with all kinds of interference factors quickly and effectively in the face of complex and changeable handling conditions, which leads to the decline of handling accuracy.

5. Conclusion

For a dual-arm robot, the synergy and equalisation of the joints of the dual-arm in its operation directly affects the smoothness and efficiency of its operation. To this end, this study considers the joint angular acceleration constraints as the premise and designs an adaptive cooperative equalisation control method for the dual-arm robot, constructing a composite controller by selecting a sliding mode variable structure controller and an adaptive fuzzy compensation controller, constructing an adaptive fuzzy model of the dual-arm robot based on the dual-arm robot dynamics model and the angular acceleration constraints, and setting a composite control law for the composite controller. Under this control law, the adaptive fuzzy model of the dual-arm robot is controlled by the composite controller to implement the adaptive synergistic equilibrium control, so that the joints of the dual-arm robot can consistently meet the set joint angular acceleration constraints under the conventional handling and vibration interference handling scenarios and realise the accurate tracking of the set trajectories of the dual-arm handling to ensure the synergistic equilibrium of trajectories of the dual-arm handling and to achieve synergistic handling. The proposed method can be adapted to the cooperative equilibrium control of a robot in different scenarios to ensure that the robot can realise efficient and smooth operation.

Looking forward to the future work, there are many valuable directions to explore. At the level of algorithm optimisation, although the current research has made some achievements, there is still room for improvement. On the one hand, more advanced optimisation algorithms can be deeply studied to optimise the parameter adjustment mechanism of the adaptive fuzzy model, such as introducing reinforcement learning algorithm, so that the model can adapt to the complex and changeable task environment and robot state more intelligently and accurately, and further improve the control accuracy and response speed. On the other hand, aiming at the chattering problem in sliding mode variable structure control, a new reaching law design method can be explored, which can effectively weaken the chattering phenomenon and improve the stability and reliability of the system while ensuring the robustness of the system.

From the perspective of system expansion, this research mainly focuses on the cooperative balanced control of dual-arm robots at present, and this method can be considered to be extended to multi-robot cooperative systems in the future. This needs to solve the key problems such as communication coordination, task allocation and collision avoidance among multi-robots. By developing efficient distributed control algorithm, the cooperative operation of multi-robot system can be realised, and the working efficiency and flexibility of the whole system can be improved to meet the practical needs of multi-robot cooperative operation on large and complex production lines.

In terms of practical application scenarios, although this method shows good performance in theory and experiment, there is still a certain gap from large-scale practical application. In the future, we should actively explore the application of this control method in more complex industrial scenes, such as working in extreme environments such as high temperature, high pressure and strong electromagnetic interference, and carrying out practical verification and optimisation in tasks that require high accuracy and stability of robot operation, such as high-precision assembly and dangerous goods handling. Through the in-depth combination with practical application scenarios, the control method is further tested and improved, and the wide application of dual-arm robots in industrial fields is promoted.

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Biographies

Guichao Cai received the master's degree in industrial engineering from North China Electric Power University in 2014. He currently works as an Engineer with the Chaozhou Power Supply Bureau Guangdong Power Grid Co., Ltd. His main research directions include electrical engineering and automation.

Yutong Zeng received the bachelor's degree in electrical engineering and automation from Guangdong Polytechnic Normal University in 2020. He currently works as an Assistant Engineer with the Substation Management Department, Chaozhou Power Supply Bureau Guangdong Power Grid Co., Ltd. His primary research focus includes power electronics technology.

Yuzhang She received the bachelor's degree in communication engineering from South China Normal University in 2019. He currently works as an Engineer with the Chaozhou Power Supply Bureau Guangdong Power Grid Co., Ltd. His main research areas include power system planning and operation.

Haocong Zheng graduated with a major in Power System and Its Automation from Guangdong University of Technology in 2017. He currently works as an Engineer with the Substation Management Department, Chaozhou Power Supply Bureau Guangdong Power Grid Co., Ltd. His primary research direction includes electric motor drive technology.

Yingxuan Zhuang received the bachelor's degree in power engineering and automation from Guangdong University of Technology in 2001. She currently works as a Senior Engineer with the Chaozhou Power Supply Bureau Guangdong Power Grid Co., Ltd. Her main research areas include power system planning and operation.