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A NOVEL INTELLIGENT FAULT OBSERVER TO DIAGNOSE ACTUATOR FAULT AND SENSOR NOISE BASED ON PROBABILITY DISTRIBUTIONS

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Abstract

This paper aims to bridge the gap between classic model base observer and intelligent observer based on the model base and probability distributions to monitor the plant which is affected by actuator fault and sensor noise simultaneously. The paper includes the implementing and designing of two new optimal actuator observers which also relies on two optimal theories. The first new optimal actuator fault observer is a model base observer based on Lyapunov conditions while, the novelty of the second optimal actuator observer comes from introducing a fault diagnosis algorithm based on a sequential sampling filter (SSF) as a prominent part of the algorithm to filter the diagnosed fault in using the rules of the algorithm. The filters depend on the likelihood ratio, which consists of two hypothetical distributions; the target distribution and the sampling distribution. The new intelligent observer has given rise to innovative algorithms in the field of fault detection and diagnose observer. Moreover, the algorithms verify Lyapunov conditions. In addition, to demonstrate the integrity of the observers, a comparison has been made in the presence of white sensor noise, which is the main reason for inaccurate measurement in addition to two types of actuator faults; Gaussian and non-Gaussian. The study used to verify the performance, where two physical and relative error criteria have been used to evaluate the performance. However, the simulation results appear the flexibility of the second observer to diagnose the actuator fault with sensor noise together due to the number of parameters in diagnosing rules and parameters of the two probabilities.

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Key Words

Fault tolerant control (FTC), fault prognosis (FP), optimal actuator fault diagnosis observer (AFO), intelligent optimal actuator fault diagnosis observer (IAFO), sequential sampling filter (SSF)

1. Introduction

The research in fault tolerant control (FTC) and prognosis has gotten attention due to the significant complexities in manufacturing industrial systems with the height maintenance cost. The main strategies of FTC and prognosis are based on implementing fault detection and diagnosis observers where there are benefit points, such as; identifying and estimating the magnitude of the fault that has still not been catered for, thus predicting new faults, reducing the risk associated with fault launching new ideas, helps the controller learn faster to overcome the fail and introduces innovative solutions rather than adding more to known ones.

For decades, researchers have been trying to introduce new observation algorithms. The classification of fault and noise has expanded the field of knowledge, where Gaussian and non-Gaussian fault and noise have been mainly considered in dynamic systems. In addition, the modification of the observers depends on the type of location of the fault where the additive observers deal with the parameters faults [1]–[10] whereas actuator observers are organised to deal with the fault in inputs and shed light on the impact of the observers [11]–[19].

Many researchers in the automation field have been attracted to fault-tolerant control as a critical strategy for designing the controller to overcome failures or sudden shutdowns of systems [20]–[22].

The design of a controller possesses integrity with a varied or fixed suitably to guarantee satisfactory performance [23]. Some researchers prefer to use the term FTC [21], [24] whereas others prefer reliable control [25], [26].

However, to avoid failure or unnecessary maintenance, FP in machines has been the goal of many papers. The

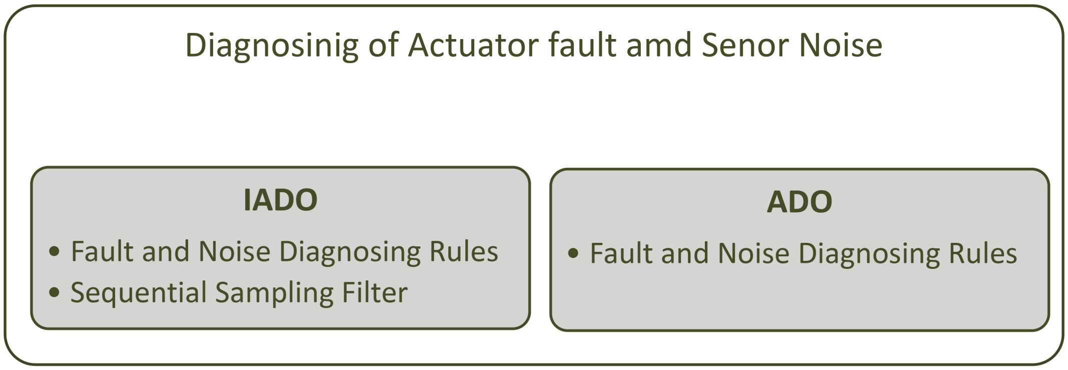

Figure 1. Diagnosing of actuator faults and sensor noise.

observers used to estimate the next stage of the machine faults in addition to the stochastic algorithm [20], [22], [27]–[34] while [35] introduced a new fuzzy and sequential important sampling filter to implement a new nonlinear observer.

Wang and Winters [36] established a methodology for prognosis using neuro-fuzzy techniques for recurrent dynamic systems. Adams and Nataraju [37] found a nonlinear dynamics framework to prognosis the faults by developing an analytically sound means for extracting features.

Al-Bayati and Wang [38] introduced three linear actuator fault observers to observe the plant that suffered an actuator fault but without sensor noise. The diagnosed fault was an actuator fault for two observers, while the diagnosed fault was an additive fault for the third observer. Observers relied on three optimal theories where the diagnosed fault the first and third observers was based on the Lyapunov function with two positive definite matrices. In contrast, the second observer implied three positive definite matrices. This paper is an extension of the research in [38] which presents two new types of actuator fault observers to deal with an actuator fault in the prescience of sensor noise which also relies on two optimal theories. The objective of this paper how to design an observer can estimate the actuator fault and sensor noise in the real plant simultaneously optimal actuator fault diagnosis observer (AFO) and intelligent optimal actuator fault diagnosis observer (IAFO). Thus, the observer (AFO) is designed to be an optimal observer and includes fault detection and diagnosis rules based on pre-specified gain matrices to achieve two optimal Lyapunov conditions. At the same time, the observer (IAFO) has been supported with a novel intelligent fault algorithm based on fault detection and diagnosis rules and a sequential sampling filter (SSF). In addition, the rules achieve two optimal Lyapunov conditions, whereas the filer depends on selecting two appropriate distributions, a significant distribution and a nominal distribution, on obtaining a high likelihood ratio.

However, the structure of writing the paper is as follows: Section 2 presents the design of a new optimal actuator fault observer (AFO) whereas the criteria to evaluate the performance of the proposed observers are introduce in Section 3. Moreover, Section 4 demonstrates the proposed two optimal observers. Finally, the conclusion and discussion for the obtained results included in Section 5 while the proving of the two optimal theorems supposed in Appendix.

2. Design a New Optimal Actuator Fault Diagnosis Observers

To design an observer to diagnose the actuator fault and sensor noise in the real time system together, two new observers ADO and IADO as shown in Fig. 1 have been introduced. The observer ADO based on rules with two definite matrices while IADO depends on SSF and rules based on one definite matrix. Consequently, the rules in IADO has single definite matrix and the parameters of the distributions in the filter. The implantation of the SSF consists of the following points:

1. Likelihood ratio which is a ratio of two distributions; target distribution and sampling distribution.
2. The likelihood ratio is used to be a weight of a considered function.
3. The weights are normalised or unnormalised.

2.1 The Model of the System

The model of the system in [38] based effected only by the actuator fault while in this paper the model of the discrete linear system has been expressed to deal with hybrid undesirable effects that are an actuator fault and sensor noise as follows:

$$x(k+1) = Ax(k) + BL_f f(k)\, u(k) \tag{1}$$

The immeasurable states is $x(k) \in R^{n_a}$ while $u(k) \in R^{n_{\text{in}}}$ is a vector of the inputs n_{in}. $y(k) \in R^p$ denotes the measurable outputs vector p where known constant matrices $A \in R^{n_a \times n_a}, B \in R^{n_a \times n_{\text{in}}}, C \in R^{p \times n_a}$. However, $f(k)$ represents the actuator fault vector and the sensor noise $s(k) \in R^p$ has been represented as an additional vector.

2.2 Design a New Intelligent Optimal Actuator Fault Diagnosis Observer (IAFO)

The observer shown in Fig. 2 has an ideal diagnostic algorithm verified in Theorem 1 where Lyapunov function

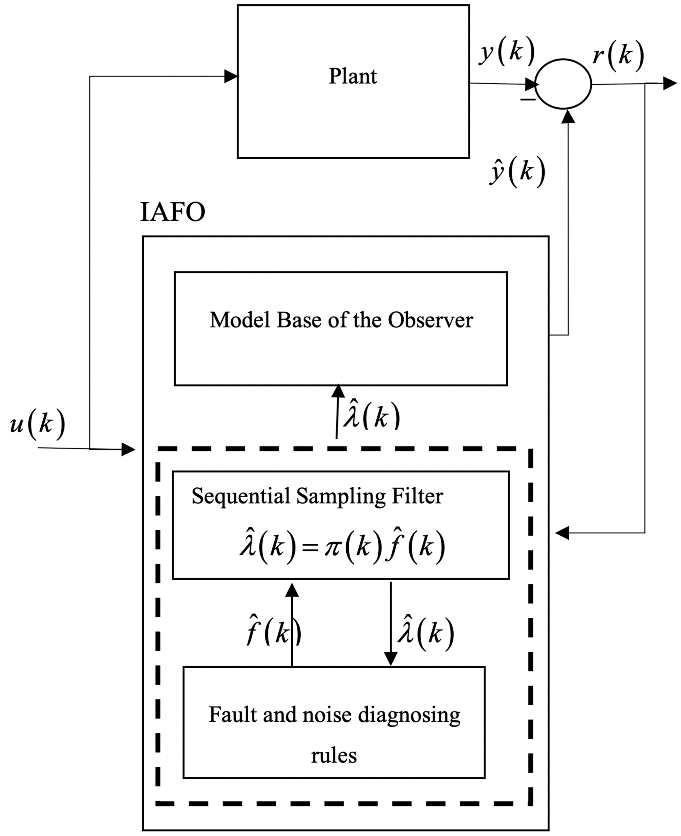

Figure 2. The block diagram of the intelligent optimal actuator fault diagnosis observer (IAFO).

based on the SSF has been supposed. The discrete linear model for actuator fault observers is proposed as follows

$$\widehat{x}(k+1) = A\widehat{x}(k) + B\widehat{\lambda}(k)u(k) + H(y(k) - \widehat{y}(k)) \quad (2)$$

Where the states of the observer is $\widehat{x}(k) \in R^{n_a}$ and $\widehat{y}(k) \in R^p$ denotes the measurable outputs p vector of the observer. To important matrices are; the filtered fault $\widehat{\lambda}(k) \in R^{r_{\text{in}}}$ and the gain matrix of the observer $H \in R^{n_a \times n_a}$ which verifies the Lyapunov condition. The residual $r(k) \in R^p$ which is used to evaluate the performance is represented as follows

$$r(k) = y(k) - \widehat{y}(k) \quad (3)$$

2.2.1 Design of a Sequential Sampling Filter (SSF) for IADO

In this paper, the SSF has been used where weights are unnormalised. To filter the fault, the expectation of the diagnosed fault $\widehat{f}(k)$ is written as follows:

$$E\left[\widehat{f}(k)\right] \simeq \frac{1}{N_s}\sum_{k=1}^{N_s} P_k\left(\widehat{f}(k)\right)\widehat{f}(k) \quad (4)$$

where $P_k\left(\widehat{f}(k)\right)$ is a distribution that calculates the expected value of the error, but due to the complexity of the distribution, two distributions have been proposed; the target distribution $\delta(k)$ and the sampling distribution $\beta(k)$, which draws samples. So the probability $P_k\left(\widehat{f_k}\right)$ will be rewritten as [39].

$$\pi(k) = P_k\left(\widehat{f_k}\right) = \frac{\delta(k)}{\beta(k)} \quad (5)$$

This is also called the likelihood ratio. Distribution $\beta(k)$ is the distribution of importance and $\delta(k)$ is the nominal distribution. Furthermore, importance sampling gives high recognition when it works well through the of a good selection of distributions' parameters. Hence, the distribution $\pi(k)$ is considered weight as follows:

$$\pi(k) = \frac{\delta(k)}{\beta(k)} \quad (6)$$

$$\pi(k) = \frac{\delta(k)/\beta(k)}{\sum_{k=1}^{N_s}\delta(k)/\beta(k)} \quad (7)$$

Furthermore, the expectation will be as follows:

$$E\left[\widehat{f}(k)\right] \simeq \frac{1}{N_s}\sum_{k=1}^{N_s}\frac{\delta(k)}{\beta(k)}\widehat{f}(k) \quad (8)$$

Distributions will be rounded for normalisation [40]:

$$\delta(k) = \frac{1}{Z_1}\widehat{\delta}(k) \quad (9)$$

In addition, the importance of weight $\widehat{\pi}(k)$ has been redefined to be as follows:

$$\widehat{\pi}(k) = \frac{Z_2}{Z_1}\frac{\widehat{\delta}(k)}{\widehat{\beta}(k)} \quad (10)$$

$$\frac{Z_2}{Z_1} = \sum_{k=1}^{N_s} \frac{\widehat{\delta}(k)}{\widehat{\beta}(k)} \beta(k) \tag{11}$$

$$\frac{Z_2}{Z_1} \simeq \frac{1}{N_s} \sum_{k=1}^{N_s} \frac{\widehat{\delta}(k)}{\widehat{\beta}(k)} \tag{12}$$

The probability $\beta(k)$ has been approximated twice: once for the main term and once to normalise the importance weights as follows:

$$\widehat{E}\left[\widehat{f}(k)\right] = \frac{1}{N_s} \sum_{k=1}^{N_s} \widehat{\pi}(k) \widehat{f}(k) \tag{13}$$

However, there is no guarantee that $\beta(k)$ or $\delta(k)$ will be normalised, so the second form of approximation (13) assumes both are normal and will not be equal to the original in expectation.

$$\mu\left\{E\left[\widehat{f}(k)\right] - \widehat{E}_k\left[\widehat{f}(k)\right]\right\} = 0 \tag{14}$$

The expected variance will be reduced using the importance sampling method when the sample count is increased [39]. As quantified in [41], the asymptotic variance σ_f^2 will be as follows [42], [43]:

$$\text{variance}\left(\widehat{E}\left[\widehat{f}(k)\right]\right) = \frac{\sigma_f^2}{N_s} \tag{15}$$

For further simplification

$$\text{variance}\left(\widehat{E}\left[\widehat{f}(k)\right]\right) = \frac{1}{N_s}\left(\sum_{k=1}^{N_s}\left(\frac{\left(\widehat{f}(k)\right)^2 (\delta(k))^2}{\beta(k)}\right) - \left(E\left[\widehat{f}(k)\right]\right)^2\right) \tag{16}$$

It is safe with protection from its failures. Suppose the numerator in (16) goes to zero slower $\beta(k)$, which means that variance $\left(\widehat{E}[f(k)]\right) \to \infty$. By incorporating these expressions into this research paper, we can enhance the filter effect on fault diagnosis.

However, they should be used in appropriate contexts where the important sampling method faces one of the biggest problems; poor sampling distribution $\beta(k)$ which means poor parameter selection of sample distribution leads to an estimate of large variability $\widehat{E}\left[\widehat{f}(k)\right]$ [40].

A non-zero sample distribution was chosen to overcome the problem, which is also $\delta(k)$ selected non-zero support by reducing the $\alpha-$ divergence as following:

$$D(\delta(k)\,||\beta(k)) = \frac{4}{1-\alpha^2}\left(1 - \sum_{k=1}^{N_s}\left((\delta(k))^{\frac{(1+\alpha)}{2}} (\beta(k))^{\frac{(1-\alpha)}{2}}\right)\right) \tag{17}$$

Zero is avoided in this range and will usually choose a $\beta(k)$ that covers $\delta(k)$. So choose the sampling distribution as in rejection sampling to prevent the 'blow up', where $\delta(k) \leq \mathrm{K}\beta(k)$ and K is constant.

Furthermore, the effective sample size ASS_N is used to evaluate the impact on the simulation variance of increasing sample size depending on the weight. Therefore, an appropriate sample size ASS_N is used to assess the effect on simulation variance to increase the sample size depending on weight in (13) [44].

$$\mathrm{ASS}_N = \frac{N_s}{1 + \text{variance}(\widehat{\pi}(k))} \tag{18}$$

2.2.2 Fault and Noise Diagnosing Rules for IADO

The concept of the algorithm is based on the filtering of the diagnosed fault $\widehat{f}(k) \in R^{r_{\text{in}}}$ using the sequential sampling algorithm [39]. The filter uses likelihood ratio in (5). Thus, the new adaptive diagnostic rules for obtaining the filtered fault can be formalised as follows:

$$\begin{aligned}\widehat{\lambda}(k) &= \pi(k)\,\widehat{f}(k)\,k > k_f \widehat{f}(k+1) \\ &= -\Gamma\widehat{\lambda}(k)\,\widetilde{\lambda}(k) = (f(k) + s(k)) - \widehat{\lambda}(k)\;\widetilde{\lambda}(k) \to 0\end{aligned} \tag{19}$$

If there is no fault in the plant; $\widehat{f}(k)$ is a set of $\widehat{f}_k^H = 1^{n_{\text{in}}\times 1}$ them which $\widehat{\lambda}(k)$ is a set of $\widehat{\lambda}(k) = 1^{n_{\text{in}}\times 1}$ while $\left(\Gamma = \Gamma^T\right)$ is proposed pre-specified gain and $\pi(k)$ is the likelihood ratio in (5). So, the dynamic error $e(k+1)$ has been calculated as follows:

$$e(k+1) = \widehat{x}(k+1) - x(k+1) \tag{20}$$

Substituting the algorithm with error in (37), the error will be as follows:

$$e(k+1) = A\widehat{x}(k) + B\pi(k)\,\widehat{\lambda}(k)\,u(k) \tag{21}$$

It can also represent as

$$\begin{aligned}e(k+1) &= (A - HC)\widehat{x}(k) + Hy(k) + Hs(k) \\ &\quad -\mathrm{A}\mathrm{x}(k) + B\widetilde{\lambda}(k)\,u(k)\end{aligned} \tag{22}$$

Using the assumption that $\bar{A} = A - \mathrm{HC}$, the dynamic error can be expressed as follows.

$$e(k+1) = \bar{A}e(k) + Hs(k) + B\widetilde{\lambda}(k)\,u(k) \tag{23}$$

Theorem 1: *Suppose that the gain matrix of the adaptive observer can be obtained by satisfying the following conditions.*

$$\bar{A}P\bar{A}^T - P = -Q_1 \tag{24}$$

where $P = P^T, Q_1 = Q_1^T$ *and* $Q_2 = Q_2^T$ *are the positive definite,* $\bar{A}$ *is a Hurwitz matrix and the* $(\bar{A}, B)$ *is completely controllable.*

2.3 Design a New Optimal Actuator Fault Diagnosis Observer (AFO)

The observer ADO as shown in Fig. 1 breaks the mould by proposing a new an optimal diagnose algorithm as in Theorem 2. In addition, The model of the observer for the plant in (1) has been expressed as follows:

$$\widehat{x}(k+1) = A\widehat{x}(k) + B\widehat{f}(k)\,u(k) + H(y(k) - \widehat{y}(k)) \tag{25}$$

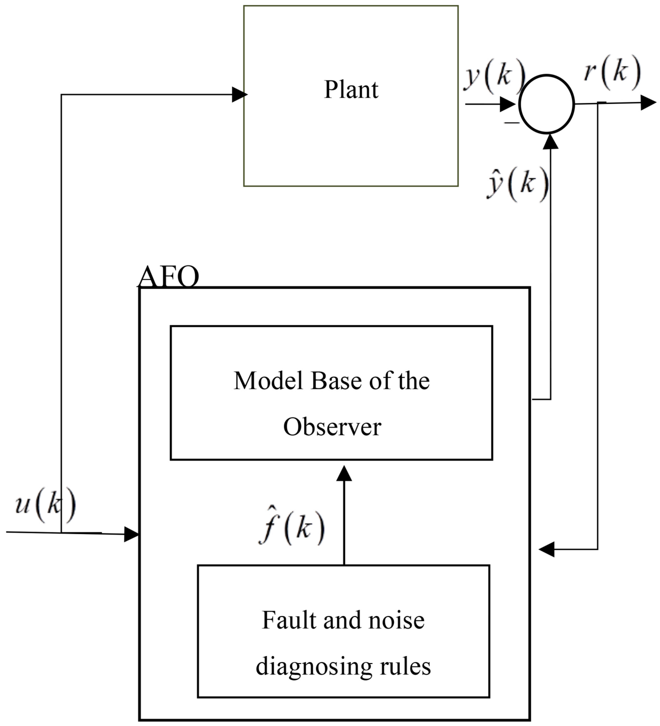

Figure 3. The block diagram of the new optimal actuator fault diagnosis observer (AFO).

The gain matrix $H \in R^{n_a \times n_a}$ achieves the Lyapunov condition while $\widehat{x}(k) \in R^{n_a}$ is the states of the observer. $\widehat{y}(k) \in R^p$ denotes the measurable outputs p vector for the observer.

2.3.1 Fault and Noise Diagnosing Rules for ADO

The fault diagnosis algorithm has been designed to detect and diagnose the fault $\widehat{f}(k)$ by selecting an observer gain vector H based on following rules for the adaptive diagnostic algorithm:

$$r(k) \to 0 \tag{26}$$

$$\begin{aligned}\widetilde{f}(k) &= \widehat{f}(k) - (L_f f(k) + s(k))\widehat{f}(k+1) \\ &= -\Gamma_1 \widehat{f}(k) - \Gamma_2 r(k)\, u(k)\, k > k_f \end{aligned} \tag{27}$$

where $\Gamma_1 = \Gamma_1^T$ and $\Gamma_2 = \Gamma_2^T$ which are pre-specified gain matrices. $f_H(k) = 1^{r \times 1}$ until a fault is detected.

Theorem 2: *The adaptive observer in* (2) *can be obtained by verifying the following conditions:*

$$\bar{A} P \bar{A}^T - P = -Q_1 \tag{28}$$

Where $P = P^T, Q_1 = Q_1^T, Q_2 = Q_2^T$ *are the positive definite and* $\bar{A}$ *is a Hurwitz matrix and the pair* $(\bar{A}, B)$ *is entirely controllable* [38].

3. Performance Evaluation

Evaluating the performance based on error and calculated criteria for relative error. The amount of physical error in the prediction is absolute, while the relative error shows how the estimates relate to the magnitude of the diagnosed fault. Some statistical criteria used as root mean squared error (r_y), mean absolute error (m_y), and variance absolute error (v_y) utilise for absolute error.

While a correlation coefficient (c_y) has been found for relative error where the expression $(\bar{y}, \bar{\widehat{y}})$ represent the mean of the plant output (y) and the mean of the observer output $(\widehat{y})$, respectively.

$$r_y = \frac{1}{N_s} \sqrt{\sum_{k=1}^{N_s} \left(\widehat{y}(k) - y(k)\right)}^2 \tag{29}$$

$$m_y = \frac{1}{N_s} \sum_{k=1}^{N_s} \left|\widehat{y}(k) - y(k)\right| \tag{30}$$

$$v_y = \frac{1}{N_s} \sum_{k=1}^{N_s} \left(\widehat{y}(k) - y(k) - m_y\right) \tag{31}$$

$$c_y = \frac{\sum_{k=1}^{N_s} \left(y(k) - \bar{y}\right)\left(\widehat{y}(k) - \bar{\widehat{y}}\right)}{\sqrt{\sum_{k=1}^{N_s} \left(y(k) - \bar{y}\right)^2 \sum_{k=1}^{N_s} \left(\widehat{y}(k) - \bar{\widehat{y}}\right)^2}} \tag{32}$$

4. Case Study and Results

In this section, a DC servo motor is used to study the factors and success of algorithms to deal with both types of faults (Gaussian and non-Gaussian) in the system in the presence of sensor noise. DC servo motor is a second-class system with multiple inputs and multiple outputs. The continuous time model of a DC motor without fault or noise as a state space form can be expressed as follows [6], [38].

$$\begin{aligned}\dot{x}(t) &= \begin{bmatrix} \dot{I}_A(t) \\ \dot{\omega}(t) \end{bmatrix} = \begin{bmatrix} -a_r/L_a & -m_f/a_l \\ m_f/I_c & -v_f/I_c \end{bmatrix} \begin{bmatrix} I_A(t) \\ \omega(t) \end{bmatrix} \\ &+ \begin{bmatrix} 1/a_l & 0 \\ 0 & -1/I_c \end{bmatrix} \begin{bmatrix} a_v(t) \\ T_l(t) \end{bmatrix}\end{aligned} \tag{33}$$

The continuous time system [45] can be discretised using the sampling time of 0.1 s to obtain the discrete-time model in the presence of fault and noise as follows:

$$\begin{aligned}\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} &= \begin{bmatrix} -0.0120 & -0.1535 \\ 0.0545 & 0.6949 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} \\ &+ \left(\begin{bmatrix} 0.19 & 0.02 \\ .019 & -0.02 \end{bmatrix} \begin{bmatrix} f_1(k) \\ f_2(k) \end{bmatrix} \begin{bmatrix} 0.4661 & 0.9199 \\ 0.9199 & -4.4021 \end{bmatrix} \begin{bmatrix} u_1(k) \\ u_2(k) \end{bmatrix} \right)\end{aligned} \tag{34}$$

The likelihood ratio (5) assumed that it was based on the target distribution $\delta(k)$ that use the Chi distribution (with an non-integer DOF parameter; DOF represents the mean of the Chi-square) where $K_\delta = 0.4$.

$$\delta(k) = \left(\widehat{f}(k)\right)^{(K_\delta - 1)} e^{-\frac{\left(\widehat{f}(k)\right)^2}{2}}, \widehat{f}(k) \geq 0 \tag{35}$$

While the sample distribution $\beta(k)$ used Gaussian distribution which can be expressed as following:

$$\beta_k = \frac{k_c}{\sqrt{2\pi\sigma_\beta}} e^{-\left(\frac{\mu_\beta - \widehat{f}(k)}{2\sigma_\beta}\right)^2}, \widehat{f}(k) \geq 0 \tag{36}$$

Table 1
The Performance for the First Outputs of the Observers IAFO and AFO

Fault type	Observer type	r_y	m_y	v_y	c_y
White fault Mean = 0, variance = 0.5 (White sensor noise)	IAFO	2.305865	1.100561	-0.957368	-0.034569
	AFO	6.025085	1.784024	-1.7737276	-0.008784
Coloured fault Mean = 0.5, Variance = 2 (White sensor noise)	IAFO	2.202556	1.103885	-1.2326292	-0.033700
	AFO	6.381026	1.853089	-1.9046964	-0.091624
Non-Gaussian fault [0.01 + 0.6 sin(x)] (White sensor noise)	IAFO	1.887358	0.997372	-0.9471871	0.01091208
	AFO	5.358581	1.627397	-1.6312927	-0.01879913

Table 2
The Performance for the Second Outputs of the Observers IAFO and AFO

Fault type	Observer type	r_y	m_y	v_y	c_y
White fault Mean=0, Variance = 1.5 (White sensor noise)	IAFO	1.142485	0.773330	-0.76319	0.4389157
	AFO	11.63176	2.515447	-2.66237	0.1145048
Coloured fault Mean=0.5, Variance = 2 (White sensor noise)	IAFO	2.202556	1.103885	-1.23262	-0.033700
	AFO	6.381026	1.853089	-1.90469	-0.091624
Non-Gaussian fault [0.01 + 0.6 sin(x)] (White sensor noise)	IAFO	1.0095794	0.7123865	-0.71282	0.45705471
	AFO	10.676883	2.3186366	-2.38544	0.08061537

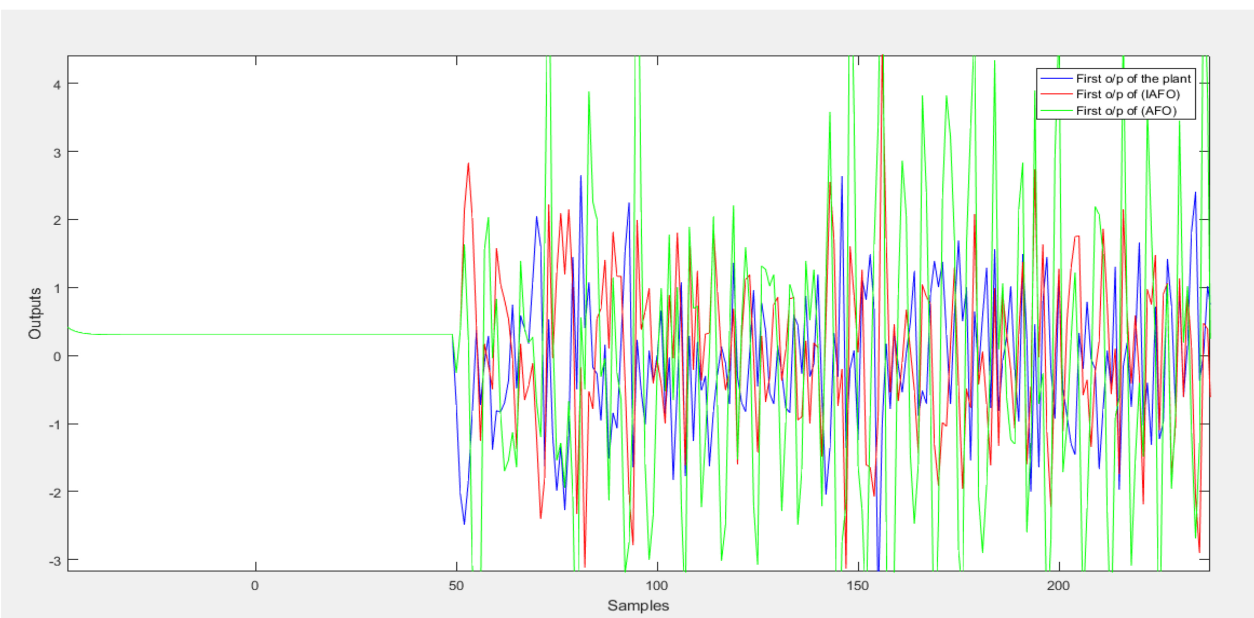

Figure 4. The first outputs of the plant, IAFO and AFO (white fault) - with white sensor noise.

where $k_c = 5.8$ are the Chi DOF parameter, multiplicative constant to make, $\delta_k < k_c\beta_k.$ $, \sigma_\beta = \sqrt{1.29}, \mu_\beta = 0$ represent standard deviation and mean, respectively. To study the observers' activity; different types of actuators faults in presence of sensor noise as follows:

1) The sensor noise is assumed to be white noise (variance = 1 and mean = 0).
2) Three types of actuator faults are applied to the system as follows:
   a. White fault (Mean = 0, variance = 0.5).

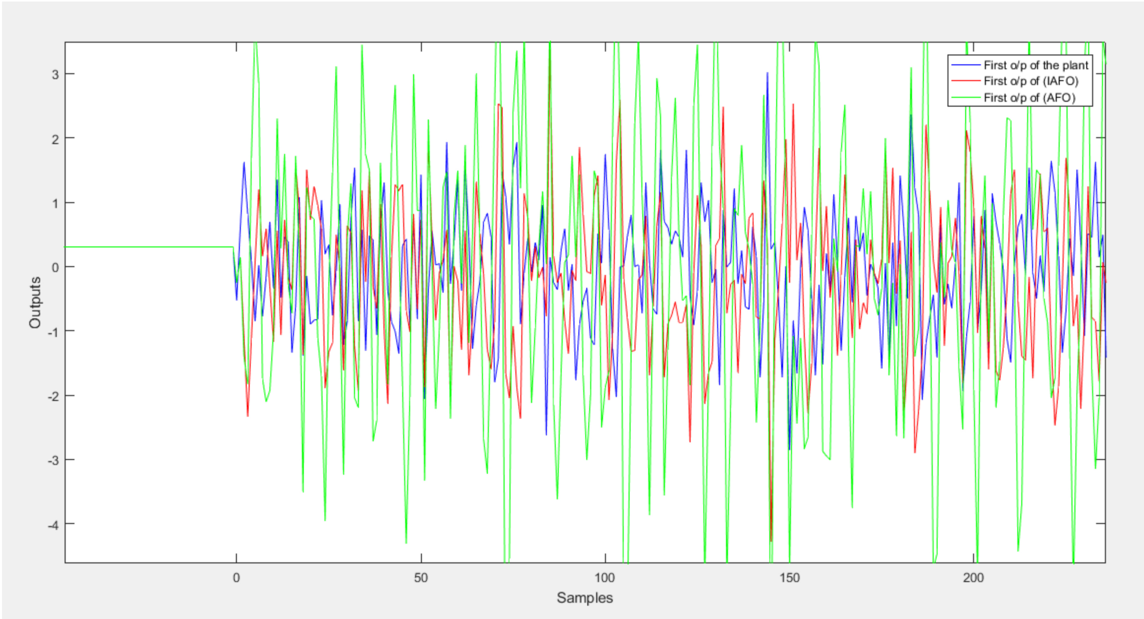

Figure 5. The first outputs of the plant, IAFO and AFO (coloured fault) - white sensor noise.

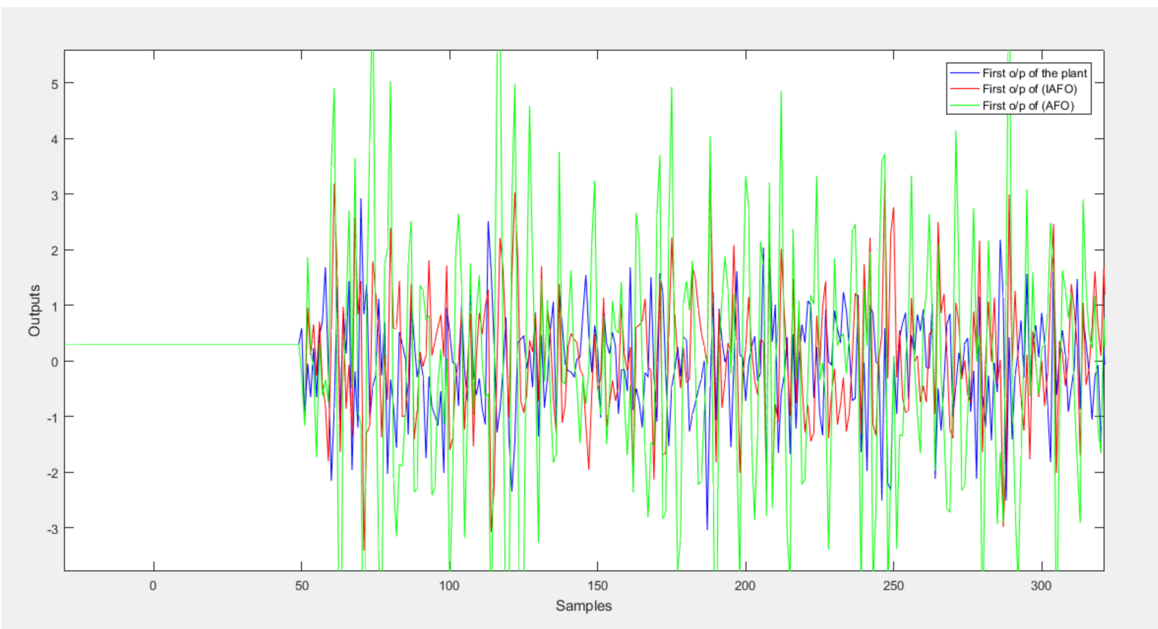

Figure 6. The first outputs of the plant, IAFO and AFO (non-Gaussian fault) - white sensor noise.

b. Coloured fault (Mean = 0.5, Variance = 2).
c. Non-Gaussian fault [0.01 + 0.6 sin(x)].

The results of performance criteria have been studied to verify the effectiveness of each observer design. Table 1 shows the values of performance criteria for the first output. In contrast, Table 2 demonstrates the performance for the second output.

Moreover, Figs. 4–6 appear the first output for the plant, the observers (IAFO) and (AFO) where the diagnosing of first outputs of IADO are closer to the plant while the ADO has more gap in diagnosing.

In contrast, Figs. 7–9 show the diagnosing of second outputs for IAFO are better than AFO in the presence the sensor noise and three types of actuator fault; white fault, coloured fault, and non-Gaussian fault, respectively.

In comparison, the results shown in the tables and figures reflect the novelty of the intelligent observer (IAFO), which is better than the observer (AFO). The high efficiency and flexibility of (IAFO) are because of the fault detection algorithm of (IAFO) which has been developed to be more flexible due to containing more rules and parameters. Hence, this research paves the way for future progress in the use of filters within the fault detection algorithm through the filters using other types of probabilities.

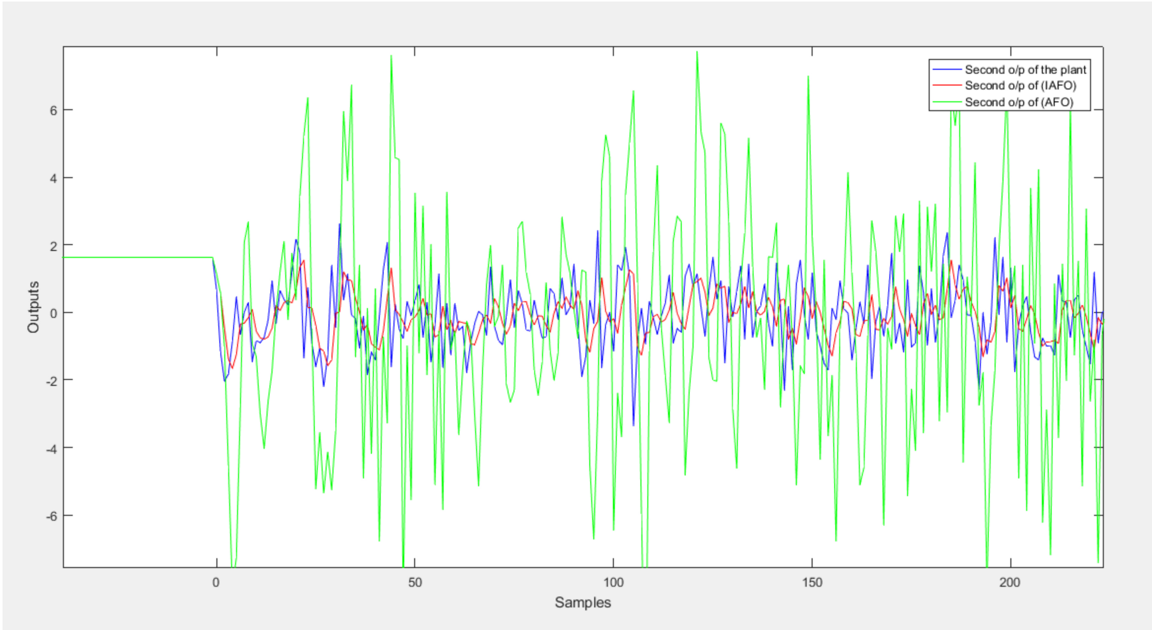

Figure 7. The second outputs of the plant, IAFO and AFO (white fault) - with white sensor noise.

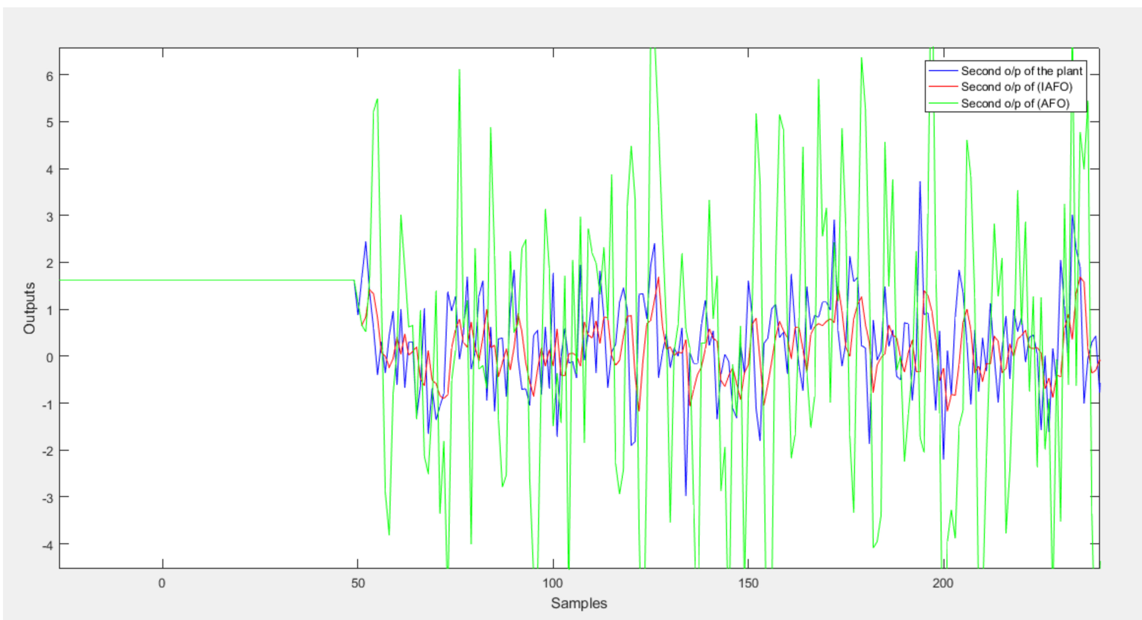

Figure 8. The second outputs of the plant, IAFO and AFO (coloured actuator fault) - white sensor noise.

5. Conclusion

In the realm of actuator fault detection and diagnosis observer, this paper presents a new idea for fault diagnosing observes where two types of actuator fault observers have been introduced which also relies on two optimal theories, assuming the plant affected by the sensor noise and actuator fault in the same time. The new AFO implies diagnosing of both an actuator fault and sensor noise simultaneously. While the new IAFO is an intelligent type which includes diagnosing rules that match the proposed Lyapunov conditions and SSF. In addition, the SSF is based on the likelihood ratio depending on two distributions; target distribution and sampling distribution. The rules of the two observers involve obtaining matrices to verify the proposed Lyapunov conditions.

Furthermore, proper selection of a good distributions is critical to improving the observer score (IAFO). At the same time, to evaluate the performance of observers, sensor noise is considered white noise, while three types of actuator faults have been applied on multi-input and multi-output system to evaluate the performance of observers: white noise, coloured noise, and non-Gaussian error. Two physical and relative error criteria have been considered performance parameters; root mean square error, average absolute error, absolute error of variance,

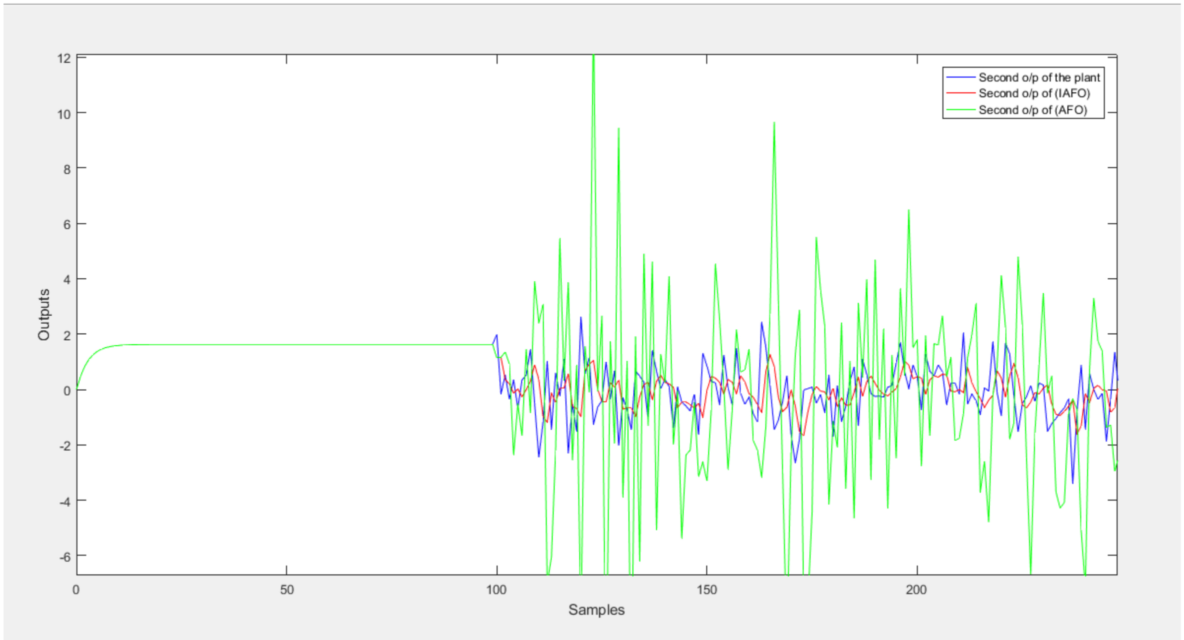

Figure 9. The second outputs of the plant, IAFO and AFO (non-Gaussian fault) - white sensor noise.

and correlation coefficient. Finally, the obtained results can reap the benefits of the simulation which showed that the intelligent observer (IAFO) was superior in diagnosing actuator fault and white sensor noise compared to the observer (AFO).

Appendix

Proof of Theorem 1. Define the following Lyapunov function

$$\hbar\left(e(k),\widetilde{\lambda}(k)\right) = (e(k))^T \mathrm{Pe}(k) + \left(\widetilde{\lambda}(k)\right)^T \Gamma_1^{-1}\widetilde{\lambda}(k) \quad (37)$$

It can be written as follows

$$\Delta\hbar\left(e(k),\widetilde{\lambda}(k)\right) = \frac{1}{2}\left(\hbar(e(k+1)) - \hbar(e(k))\right) \quad (38)$$

For further declaration, it can be represented as

$$\begin{aligned}\Delta\hbar\left(e(k),\widetilde{\lambda}(k)\right) &= \frac{1}{2}(e(k+1))^T \mathrm{Pe}(k+1) \\ &- (e(k))^T \mathrm{Pe}(k) + \left(\widetilde{\lambda}(k)\right)^T \Gamma^{-1}\widetilde{\lambda}(k+1)\end{aligned} \quad (39)$$

It can also be expressed as

$$\begin{aligned}\Delta\hbar\left(e(k),\widetilde{\lambda}(k)\right) &= \frac{1}{2}\left[\bar{A}e(k) + B\widetilde{\lambda}(k)u(k)\right]^T \\ &\quad P\left[\bar{A}e(k) + B\widetilde{\lambda}(k)u(k)\right]\end{aligned} \quad (40)$$

$$\begin{aligned}\Delta\hbar\left(e(k),\widetilde{\lambda}(k)\right) &= \frac{1}{2}(e(k))^T\left[\bar{A}^T P\bar{A} - P\right]e(k) \\ &+ \frac{1}{2}\left(B\widetilde{\lambda}(k)u(k)\right)^T P\left(B\widetilde{\lambda}(k)u(k)\right)\end{aligned} \quad (41)$$

By supposing

$$\partial(k) = \left(B\widetilde{\lambda}(k)u(k)\right) \quad (42)$$

Therefore, it can be rewritten as

$$\begin{aligned}\Delta\hbar\left(e(k),\widetilde{\lambda}(k)\right) &= -\frac{1}{2}(e(k))^T Q_1 e(k) + \frac{1}{2}(\partial(k))^T P\partial(k) \\ &+ \left(\widetilde{\lambda}(k)\right)^T\left(B^T P - Q_2\right)e(k)\end{aligned} \quad (43)$$

To make the term zero or negative, u_k will be derived, and (19), (41) are substituted into (48)

$$\Delta\hbar\left(e(k),\widetilde{\lambda}(k)\right) \le \left(-\frac{1}{2}\lambda 1\left\|e(k)\right\|\left\|\Gamma\right\|\left\|\pi(k)\right\|\left\|\widehat{\lambda}(k)\right\| \frac{1}{2}\left\|P\right\|\left\|\partial(k)\right\|_{\min}\right) \quad (44)$$

Furthermore, the inequality can be expressed as

$$\Delta\hbar\left(e(k),\widetilde{\lambda}(k)\right) \le \left(-\frac{1}{2}\lambda 1\left\|e(k)\right\|\left\|\Gamma\right\|\left\|\pi(k)\right\|\left\|\widehat{\lambda}(k)\right\| \frac{1}{2}\left\|P\right\|\left\|\partial(k)\right\|_{\min}\right) \quad (45)$$

Raleigh–Ritz inequality, Cauchy–Schwartz inequality, index matrix for the first term and the norms for the second and third terms, respectively.

$$\Delta\hbar\left(e(k),\widetilde{\lambda}(k)\right) \le \left\|e(k)\right\| \left(-\frac{1}{2}\lambda 1\left\|e(k)\right\| \frac{1}{2}\frac{\left\|P\right\|\left\|\partial(k)\right\|}{\left\|e(k)\right\|}\frac{\left\|\Gamma\right\|\left\|\pi(k)\right\|\left\|\widehat{\lambda}(k)\right\|}{\left\|e(k)\right\|}_{\min}\right) \quad (46)$$

Suppose there is an existence $(\eta_1, \eta_2) > 0$ no matter how small $(\kappa_1, \kappa_2) > 0$, as follows

$$\|e(k)\| < \kappa_1 \Rightarrow \frac{\|\partial(k)\|}{\|e(k)\|} < \eta_1, \tag{47}$$

To ensure the theory of stability of Lyapunov

$$\Delta\hbar\left(e(k), \widetilde{\lambda}(k)\right) \leq \|e(k)\| \left(-\frac{1}{2}\lambda 1 \|e(k)\| \frac{1}{2}_1 \|P\|_2 \|\Gamma\| \|\pi(k)\|_{\min}\right) \tag{48}$$

Thus, the condition will be met as follows

$$\frac{\lambda 1 \|e(k)\|_{\min}}{\frac{1}{2}\|P\| + \|\Gamma\| \|\pi(k)\| (\eta_1, \eta_2)} \tag{49}$$

Proof of Theorem 2. Define the following Lyapunov function

$$v\left(e(k), \widetilde{f}(k)\right) = e^T(k) \operatorname{Pe}(k) + \widetilde{f}^T(k) \Gamma_2^{-1} \widetilde{f}(k) \tag{50}$$

It can be shown that

$$\Delta v\left(e(k), \widetilde{f}(k)\right) = \frac{1}{2}\left(v(e(k+1)) - v(e(k))\right) \tag{51}$$

It can be further expressed as

$$\begin{aligned}\Delta v\left(e(k), \widetilde{f}(k)\right) = {} & \frac{1}{2} e^T(k+1) \operatorname{Pe}(k+1) - e^T(k) \operatorname{Pe}(k) \\ & + \widetilde{f}^T(k) \Gamma_2^{-1} \widetilde{f}(k+1) \\ & + \widetilde{f}^T(k) \Gamma_2^{-1} \widehat{f}(k+1)\end{aligned} \tag{52}$$

$$\begin{aligned}\Delta v\left(e(k), \widetilde{f}(k)\right) = {} & \frac{1}{2}\left[\bar{A}e(k) + B\widetilde{f}(k) u(k)\right]^T \\ & P\left[\bar{A}e(k) + B\widetilde{f}(k) u(k)\right] \\ & - e^T(k) \operatorname{Pe}(k)\end{aligned} \tag{53}$$

For further simplification

$$\begin{aligned}\Delta v\left(e(k), \widetilde{f}(k)\right) = {} & \frac{1}{2} e^T(k) \left[\bar{A}^T P \bar{A} - P\right] e(k) \\ & + \frac{1}{2}\left(B\widetilde{f}(k) u(k)\right)^T P \left(B\widetilde{f}(k) u(k)\right) \\ & + \widetilde{f}^T(k) \operatorname{Ce}(k) + \widetilde{f}^T(k) \Gamma_2^{-1} \widetilde{f}(k+1) \\ & + \widetilde{f}^T(k) \Gamma_2^{-1} \Gamma_1 \widehat{f}(k)\end{aligned} \tag{54}$$

Due to the sensor noise

$$\begin{aligned}\Delta v\left(e(k), \widetilde{f}(k)\right) = {} & \frac{1}{2} e^T(k) \left[\bar{A}^T P \bar{A} - P\right] e(k) \\ & + \frac{1}{2}\left(B\widetilde{f}(k) u(k)\right)^T P \left(B\widetilde{f}(k) u(k)\right)\end{aligned} \tag{55}$$

By assuming $\hbar(k) = \left(B\widetilde{f}(k) u(k)\right)$, it can be rewritten as follows

$$\begin{aligned}\Delta v\left(e(k), \widetilde{f}(k)\right) = {} & -\frac{1}{2} e^T(k) Q_1 e(k) + \frac{1}{2}(\hbar(k))^T(k) P\hbar(k) \\ & + \widetilde{f}^T(k) (B^T P - Q_2) e(k) \\ & + \widetilde{f}^T(k) \operatorname{Cs}(k) + \widetilde{f}^T(k) \Gamma_2^{-1} \widehat{f}(k+1) \\ & + \widetilde{f}^T(k) \Gamma_2^{-1} \Gamma_1 \widehat{f}(k)\end{aligned} \tag{56}$$

Furthermore, by substituting (1), (2), and (5) into (12) as well as u_k would be derived to make the term zero or negative therefore, it can be as

$$\Delta v\left(e(k), \widetilde{f}(k)\right) \leq \begin{pmatrix} \& -\frac{1}{2}\lambda 1 \|e(k)\| \frac{1}{2}\|P\| \|\hbar(k)\|_{\min} \\ \& + \|C\| \|s(k)\| + \left\|\Gamma_2^{-1}\Gamma_1\right\| \left\|\widehat{f}(k)\right\| \end{pmatrix} \tag{57}$$

The Raleigh–Ritz inequality has been used for the first term whereas the Cauchy–Schwarz inequality and the index matrix norm used for the second and third terms. As result, the derivate function will be as

$$\begin{aligned}& \Delta v\left(e(k), \widetilde{f}(k)\right) \\ & \leq \|e(k)\| \begin{pmatrix} \& -\frac{1}{2}\lambda 1 \|e(k)\| \frac{\left\|\Gamma_2^{-1}\Gamma_1\right\| \|\widehat{f}(k)\|}{\|e(k)\|}_{\min} \\ \& + \frac{\|C\| \|s(k)\|}{\|e(k)\|} + \frac{1}{2}\frac{\|P\| \|\hbar(k)\|}{\|e(k)\|} \end{pmatrix}\end{aligned} \tag{58}$$

This means that for any $\gamma_1, \gamma_2, \gamma_3 > 0$, no matter how small, there is $\eta_1, \eta_2, \eta_3 > 0$, the results it is as follows

$$\begin{aligned}\|e(k)\| < \gamma_1 \Rightarrow \frac{\|\hbar(k)\|}{\|e(k)\|} < \eta_1, \|e(k)\| \\ < \gamma_2 \Rightarrow \frac{\|C\| \|s(k)\|}{\|e(k)\|} < \eta_2,\end{aligned} \tag{59}$$

For any $(\zeta_1, \zeta_2, \zeta_3) > 0$ there is $(\eta_1, \eta_2, \eta_3) > 0$ and $\|e_k\| < (\zeta_1, \zeta_2, \zeta_3)$, the inequality will be as follows

$$\begin{aligned}\Delta v\left(e(k), \widetilde{f}(k)\right) \leq \|e(k)\| \Bigg(-\frac{1}{2}\lambda 1 \|e(k)\| \frac{1}{2}_1 \|P\|_2 \|C\| \\ \|s(k)\|_3 \left\|\Gamma_2^{-1}\Gamma_1\right\|_{\min}\Bigg)\end{aligned} \tag{60}$$

To check the stability $\Delta v\left(e(k), \widetilde{f}(k)\right) < 0$ in which the linear system is asymptotically stable by the Lyapunov stability theorem, the condition that must be met is the following inequality.

$$(\eta_1, \eta_2, \eta_3) > \frac{\lambda 1 \|e(k)\|_{\min}}{\frac{1}{2}\|P\| + \left\|\Gamma_2^{-1}\Gamma_1\right\| + \|C\| \|s(k)\|} \tag{61}$$

6. Conflict of Interests

The author has no conflicts of interest to declare, and there is no financial interest to report since last five years ago.

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