NETWORKED STATE ESTIMATION OVER LOSSY COMMUNICATION CHANNELS WITH DATA RATE LIMITATION

Chunqiang Chen,*,** Weiming Tang,* Qingquan Liu,***,**** and Zhihui Dang***

Abstract

This paper deals with the state estimation problem for networked control systems. In general, data packets on plant states are transmitted over a lossy communication channel. More precisely, the system in our modelling framework is subject to both data packet dropout and data rate limitation in such a channel. To deal with the state estimation problem, we employ the differential quantization method and give a new quantization coding scheme for such systems and derive that encoders and estimators can work synchronously on the basis of the scheme. Furthermore, we further derive sufficient conditions for observability in this case. We present an illustrative example to state the effectiveness of the proposed scheme.

Key Words

Networked control systems, state estimation, data rate limitation, packet dropout, observability

1. Introduction

The problem of observability analysis of networked control systems in the presence of communication constraints has a rich, long history of literature. The phenomenon of data packet dropout and data rate limitation is rather common in such systems [1]. This has caused a great interest in observability analysis of linear systems.

The literature in this area may be divided into several parts. One part derives conditions for observability under packet dropout. Another one considers the case with data transmission delay. Furthermore, there exists some research work on observability under data rate limitation too. Packet dropout can result in loss of control performance or instability. Networked control problems under random packet losses were considered in [2]–[4]. Furthermore, the paper [5] studied the problem of state estimation for networked control systems subject to the stochastic packet delay and loss. The paper [6] and [7] addressed the state estimation problem with random time delays. A lot of excellent results on data rate limitation have been reported in the literature. Sufficient conditions on the data rate required to ensure observability and stabilization of linear control systems were provided by [8]–[11]. Data rate conditions to stabilize linear control systems were derived on the basis of event triggering [12]–[13].

Some of the classical results involving the state estimation problem for networked control systems are presented in [14]–[17]. Networked state estimation over a shared communication medium was introduced in [14]. An Linear Matrix Inequality approach could be used to design the state estimator for networked control systems [15]. A comprehensive solution to the plant state estimation problem for networked control systems in a distributed fashion over communication networks was provided in [16]. A remote state estimator over a wireless fading channel was designed in [17]. The paper [18] gave the design of an observer on the basis of sliding mode control law.

Our focus in this paper is on state estimation for networked control systems. Thus, the values of plant states need to be quantized and coded. Data packets are transmitted over a communication channel with both data packet dropout and data rate limitation. For such a channel, we will give a new quantization coding scheme, which differs from those of the literature. It guarantees that the system is controllable and observable. We will also derive that encoders and estimators can work synchronously on the basis of the scheme. Furthermore, we will further give a lower bound of data rates of the channel, above which there exists a coder-controller to ensure observability of networked control systems. The sufficient condition proposed here is derived by a new proof techniques and is less conservative. Clearly, it is a significant result.

The main contributions of this paper are as follows.

- We develop the framework for networked control systems, which are subject to both data packet dropout and data rate limitation, and deal with the state estimation problem for such systems.

* GNSS Research Center, Wuhan University, Wuhan, Hubei, 430072, China; e-mail: lqqneu@163.com
** Fujian Institute of Research on the Structure of Matter, Chinese Academy of Sciences, Fuzhou, Fujian, 350002, China
*** College of Equipment Engineering, Shenyang Ligong University, Shenyang, Liaoning, 110159, China
**** Fujian Zhongrui Network Co., LTD, Fuzhou, Fujian, 350015, China
Corresponding author: Weiming Tang

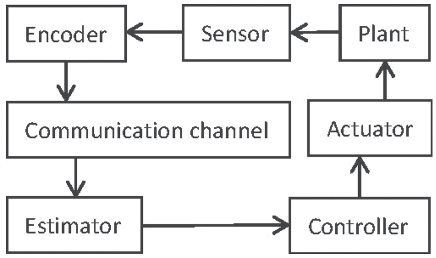

Figure 1. Networked control systems.

- We employ the differential quantization method and give a new quantization coding scheme.
- We derive that encoders and estimators can work synchronously and further derive sufficient conditions for observability in this case.

The rest of the article is organized as follows. In Section 2, we present the problem formulation. In Section 3, we give the quantization, coding, and control scheme. In Section 4, we derive sufficient conditions for observability. In Section 5, we present the results of numerical simulation. In Section 6, we state conclusions.

2. Problem Formulation

We are primarily interested in studying the following networked control system

$$X(k+1) = GX(k) + HU(k), \tag{1}$$
$$Y(k) = CX(k), k \in \mathbb{Z}^+ \tag{2}$$

where $X(k) \in \mathbb{R}^n$, $Y(k) \in \mathbb{R}^m$, and $U(k) \in \mathbb{R}^p$ denote the plant state, the measured output, and the control input, respectively. G, H, and C are known matrices. Assume that $X(0)$ is a random variable satisfying

$$\|X(0)\| \leq \varphi_0 < \infty. \tag{3}$$

Assume that the pair (G, H) is stabilizable and the pair (G, C) is observable.

As shown in Fig. 1, the encoder sends the information on the plant states to the estimator over a communication channel. First, values of plant states need to be quantized and coded. In general, data packets on plant states are transmitted over a lossy communication channel. Then, System (1) in our modelling framework is subject to both data packet dropout and data rate limitation in such a channel. Let p_d denote the packet dropout probability and let $r(k)$ denote the number of bits transmitted at the kth sampling interval. If the estimator receives no packet on plant states in the kth sampling interval, we set $\theta(k) = 0$. On the contrary, we set $\theta(k) = 1$. Clearly, $\theta(k)$ can state whether the data packet is lost.

Let $\hat{X}(k)$ and $V(k)$ denote the state estimation and the state estimation error obtained by the estimator. System (1) is said to be mean square observable if there exists a quantization coding scheme such that

$$\limsup_{k\to\infty} E\|V(k)\| \leq \phi < \infty \tag{4}$$

holds.

In this paper, we will address the state estimation problem for System (1) over such a digital communication channel with both data packet dropout and data rate limitation. The main task is to propose a new quantization coding scheme under communication constraints and to derive sufficient conditions for observability of System (1).

3. Quantization, Coding and Control Scheme

3.1 Quantization and Coding

To deal with the state estimation problem, we employ the differential quantization method and give a new quantization coding scheme for System (1). The scheme is clearly different from the existing ones in the literature. On the basis of the quantization coding scheme, we will further give a lower bound of data rates of the channel, above which there exists a coder-controller to ensure observability of System (1).

We consider the case where the system matrix G has only the real distinct eigenvalues. For other cases, there are similar results [9]. Let λ_1, λ_2, $\cdots$, λ_n denote the real distinct eigenvalues of G. Then, we may find two matrices: M and Θ such that

$$\Theta = MGM^{-1} = \text{diag}[\lambda_1, \lambda_2, \cdots, \lambda_n]. \tag{5}$$

We define

$$Z(k) := MX(k), \tag{6}$$
$$W(k) := MHU(k). \tag{7}$$

Thus, we have

$$Z(k+1) = \Theta Z(k) + W(k). \tag{8}$$

Let $Z(k) := [z_1(k) \;\; z_2(k) \;\; \cdots \;\; z_n(k)]^T$ and $W(k) := [w_1(k) \;\; w_2(k) \;\; \cdots \;\; w_n(k)]^T$. Then, we have

$$z_i(k+1) = \lambda_i z_i(k) + w_i(k), \;\; i = 1, 2, \cdots, n. \tag{9}$$

If $|\lambda_i| \leq 1$, it leads to

$$\limsup_{k\to\infty} E|v_i(k)| \leq \phi. \tag{10}$$

Next, we consider the case with $|\lambda_i| > 1$. Let $z_i(k, e)$ and $d_i(k, e)$ denote the ith state prediction and its error radius at the encoder, respectively. If one packet sent at the $(k-1)$th sampling interval is successfully delivered to the estimator at the time within $((k-1)h, kh)$ (i.e., $\theta(k-1) = 1$), the estimator can obtain the quantization value $q_i(k-1)$ at the $(k-1)$th sampling interval. Then, we may implement quantized feedback control such that

$$|z_i(k) - z_i(k, e)| \leq d_i(k, e) \tag{11}$$

holds. On the contrary, if there exists $i \in \{1, 2, \cdots, n\}$ such that

$$|z_i(k) - z_i(k, e)| > d_i(k, e) \tag{12}$$

holds, the encoder can know that the packet sent at the $(k-1)$th sampling interval is lost. In order to show the Communication State Information at the encoder, let $\alpha(k) \in \mathbb{Z}^+$ denote the number of successive packet dropout. Then, $\alpha(k)$ is given by

$$\alpha(k) = \begin{cases} 0, & \text{when } \theta(k-1) = 1 \\ \alpha(k-1)+1, & \text{when } \theta(k-1) = 0 \end{cases} \quad (13)$$

with $\alpha(0) = 0$.

Let $\hat{z}_i(k, e)$ denote the encoder estimation of $z_i(k)$. Furthermore, the encoder defines $o_i(k, e)$ as the centre of certain range of $z_i(k)$. Then, the encoder sets

$$\hat{z}_i(k-1, e) = \begin{cases} o_i(k-1, e) + q_i(k-1), & \text{when } \theta(k-1) = 1 \\ o_i(k-1, e), & \text{when } \theta(k-1) = 0 \end{cases} \quad (14)$$

We define

$$\begin{aligned} \hat{Z}(k-1, e) &= [\hat{z}_1(k-1, e) \;\; \hat{z}_2(k-1, e) \;\; \cdots \;\; \hat{z}_n(k-1, e)]^T, \\ O(k-1, e) &= [o_1(k-1, e) \;\; o_2(k-1, e) \;\; \cdots \;\; o_n(k-1, e)]^T. \end{aligned} \quad (15)$$

Let K denote the control gain. Then, the encoder sets

$$O(k, e) = (\Theta + MHKM^{-1})\hat{Z}(k-1, e) \quad (16)$$

with $O(0, e) = 0$.

We employ a uniform quantizer and divide the interval $[o_i(k, e) - |\lambda_i|^{\alpha(k)} d_i(k - \alpha(k), e), o_i(k, e) + |\lambda_i|^{\alpha(k)} d_i(k - \alpha(k), e)]$ into $m_i(k) \in \mathbb{Z}^+$ subintervals. Here, we select $m_i(k)$ satisfying

$$m_i(k) = m_{i0}^{\alpha(k)+1} > |\lambda_i|^{\alpha(k)+1}, \quad (17)$$

where m_{i0} is a positive integer larger than $|\lambda_i|$. Here, we may set $m_{i0} = \lceil |\lambda_i| \rceil$. Then, $z_i(k)$ falls into the jth one of $m_i(k)$ subintervals, $j = 0, 1, 2, \cdots, m_i(k) - 1$. The midpoint of the jth subinterval is the quantization value $q_i(k)$. It follows from [19] that the data rate $r(k) \in \mathbb{Z}^+$ must satisfy the following condition

$$\begin{aligned} r(k) &\geq (\alpha(k)+1)R_0 \\ &> (\alpha(k)+1) \sum_{i=1}^{n} \log_2 |\lambda_i| \quad \text{(bits/sample)} \end{aligned} \quad (18)$$

with $R_0 := \sum_{i=1}^{n} \log_2 m_{i0}$. Here, we may employ the minimum data rate

$$r(k) = \lceil (\alpha(k)+1)R_0 \rceil \quad \text{(bits/sample)}. \quad (19)$$

We construct a binary code $c(q_1(k), q_2(k), \cdots, q_n(k))$ with the codeword length $l(k)$, where

$$l(k) = r(k) = \lceil (\alpha(k)+1)R_0 \rceil. \quad (20)$$

Furthermore, the encoder gives the state prediction and its error radius by

$$\begin{aligned} Z(k+1, e) = (\Theta + MHKM^{-1})[&o_1(k, e) + q_1(k) \\ &o_2(k, e) + q_2(k) \;\; \cdots \;\; o_n(k, e) + q_n(k)]^T \end{aligned} \quad (21)$$

and

$$d_i(k+1, e) = \frac{|\lambda_i|}{m_{i0}} d_i(k, e). \quad (22)$$

3.2 State Estimation and Control

Let $z_i(k, s)$ and $d_i(k, s)$ denote the state prediction and its error radius at the estimator, respectively. If one packet sent at the kth sampling interval is successfully delivered to the estimator at the time within $(kh, (k+1)h)$ (*i.e.*, $\theta(k) = 1$), the estimator can obtain the codeword length $l(k)$ and compute $\alpha(k)$ by

$$\alpha(k) = \lfloor \frac{l(k)}{R_0} \rfloor - 1. \quad (23)$$

Furthermore, it can also obtain the quantization value $q_i(k)$, $i = 1, 2, \cdots, n$, at the kth sampling interval.

The estimator defines $o_i(k, s)$ as the centre of certain range of $z_i(k)$. Let $\hat{z}_i(k, s)$ denote the estimator estimation of $z_i(k)$. Then, the estimator sets

$$\hat{z}_i(k, s) = \begin{cases} o_i(k, s) + q_i(k), & \text{when } \theta(k) = 1 \\ o_i(k, s), & \text{when } \theta(k) = 0 \end{cases} \quad (24)$$

We define

$$\begin{aligned} \hat{Z}(k, s) &= [\hat{z}_1(k, s) \;\; \hat{z}_2(k, s) \;\; \cdots \;\; \hat{z}_n(k, s)]^T, \\ O(k, s) &= [o_1(k, s) \;\; o_2(k, s) \;\; \cdots \;\; o_n(k, s)]^T. \end{aligned} \quad (25)$$

Then, it follows that

$$\hat{X}(k) = M^{-1}\hat{Z}(k, s). \quad (26)$$

Here, we give

$$U(k) = K\hat{X}(k), \quad (27)$$

where all eigenvalues of $G + HK$ lie inside the unit circle.

Then, the estimator sets

$$O(k+1, e) = (\Theta + MHKM^{-1})\hat{Z}(k, s) \quad (28)$$

with $O(0, s) = 0$. Furthermore, if one packet sent at the kth sampling interval is successfully delivered to the estimator at the time within $(kh, (k+1)h)$, the estimator may obtain

$$\begin{aligned} Z(k+1, s) = (\Theta + MHKM^{-1})[&o_1(k, s) + q_1(k) \\ &o_2(k, s) + q_2(k) \;\; \cdots \;\; o_n(k, s) + q_n(k)]^T \end{aligned} \quad (29)$$

and

$$d_i(k+1, s) = \frac{|\lambda_i|}{m_{i0}} d_i(k, s). \quad (30)$$

4. Sufficient Conditions for Observability

This section derives sufficient conditions for observability of System (1) in the presence of communication constraints. To solve the state estimation problem, it is necessary that the encoder and the estimator work synchronously.

Theorem 1. *Consider System* (1). *Let R and p_d denote the average data rate of the channel and the probability of packet dropout, respectively. Then, there exists the quantization coding scheme given in Section* 3 *such that the encoder and estimator can work synchronously. System* (1) *is mean square observable if the following conditions hold:*

- p_d *needs to satisfy*

$$p_d < \frac{1}{|\lambda_i|}. \tag{31}$$

- R *needs to satisfy*

$$R > \frac{1}{(1-p_d)}\sum_{i=1}^{n}\log_2|\lambda_i| \quad \text{(bits/sample)}. \tag{32}$$

Proof. First, we assume that for any time k, we have

$$o_i(k,e) = o_i(k,s), \tag{33}$$
$$d_i(k,e) = d_i(k,s), \tag{34}$$
$$\hat{z}_i(k-1,e) = \hat{z}_i(k-1,s). \tag{35}$$

We will derive

$$\begin{aligned} o_i(k+1,e) &= o_i(k+1,s),\\ d_i(k+1,e) &= d_i(k+1,s),\\ \hat{z}_i(k,e) &= \hat{z}_i(k,s). \end{aligned} \tag{36}$$

Then, we can obtain the recursive formulation. We will also derive that $d_i(k,e)$ and $d_i(k,s)$ converge to the neighbourhoods of the origin as $k\to\infty$.

Let T_j denote the time when the jth packet is successfully delivered to the estimator. Here, we define

$$T_j = \min k \tag{37}$$

subject to

$$\begin{aligned} \theta(k) &= 1,\\ T_{j-1} &< k,\\ T_0 &= -1,\\ j,k,T_j &\in \mathbb{Z}^+. \end{aligned} \tag{38}$$

Then, we have

$$\begin{aligned} \alpha(T_j) &= T_j - T_{j-1} - 1,\\ \alpha(T_j+1) &= 0. \end{aligned} \tag{39}$$

Now, we consider the case with $\theta(T_j) = 1$ ($j = 1, 2, \cdots, \infty$). Namely, the packet sent at the T_j sampling interval is successfully delivered to the estimator at the time within $(T_j h, (T_j+1)h)$. A similar argument can be used for this case.

When the jth packet is successfully delivered to the estimator, the estimator has access to the codeword length $l(T_j)$ and can compute $\alpha(T_j)$ by

$$\alpha(T_j) = \lfloor \frac{l(T_j)}{R_0} \rfloor - 1. \tag{40}$$

Furthermore, the estimator may also obtain $q_i(T_j)$, and give

$$\hat{z}_i(T_j,s) = o_i(T_j,s) + q_i(T_j). \tag{41}$$

Then, we have

$$|\bar{v}_i(T_j)| \le \frac{1}{m_{i0}} d_i(T_j,s), \tag{42}$$

and

$$\hat{X}(T_j) = M^{-1}\hat{Z}(T_j,s). \tag{43}$$

We have

$$Z(T_j+1) = (\Theta + MHKM^{-1})\hat{Z}(T_j,s) + \Theta\bar{V}(T_j). \tag{44}$$

Furthermore, the estimator sets

$$O(T_j+1,s) = (\Theta + MHKM^{-1})\hat{Z}(T_j,s) \tag{45}$$

and

$$Z(T_j+1,s) = (\Theta + MHKM^{-1})\hat{Z}(T_j,s). \tag{46}$$

Clearly, it follows that

$$|z_i(T_j+1) - o_i(T_j+1,s)| \le \frac{|\lambda_i|}{m_{i0}} d_i(T_j,s) = d_i(T_j+1,s). \tag{47}$$

Clearly, the range of the prediction error would decrease due to $m_{i0} > |\lambda_i|$.

At the same time, the encoder sets

$$\begin{aligned} Z(T_j+1,e) = (\Theta + MHKM^{-1})[&o_1(T_j,e) + q_1(T_j)\\ o_2(T_j,e) + q_2(T_j) \quad \cdots \quad &o_n(T_j,e) + q_n(T_j)]^T. \end{aligned} \tag{48}$$

Then, it follows that

$$|z_i(T_j+1) - z_i(T_j+1,e)| \le d_i(T_j+1,e). \tag{49}$$

If the encoder finds that $z_i(T_j+1)$ satisfies the condition above, it can know that the packet sent at the T_jth sampling interval is successfully delivered to the estimator at the time within $(T_j h, (T_j+1)h)$. The encoder sets

$$\hat{z}_i(T_j,e) = o_i(T_j,e) + q_i(T_j), \tag{50}$$

and obtains

$$O(T_j+1,e) = (\Theta + MHKM^{-1})\hat{Z}(T_j,e). \tag{51}$$

Then, it follows that

$$\begin{aligned} o_i(T_j+1,e) &= o_i(T_j+1,s),\\ d_i(T_j+1,e) &= d_i(T_j+1,s),\\ \hat{z}_i(T_j,e) &= \hat{z}_i(T_j,s). \end{aligned} \tag{52}$$

For this case, the encoder and the estimator can work synchronously.

Now, we consider the case with $\theta(k) = 0$ ($T_{j-1} < k < T_j, j = 1, 2, \cdots, \infty$). For this case, the estimator may set

$$\hat{z}_i(k,s) = o_i(k,e). \tag{53}$$

Then, we have

$$|\bar{v}_i(k)| \leq |\lambda_i|^{\alpha(k)} d_i(k-\alpha(k), s), \tag{54}$$

and

$$\hat{X}(k) = M^{-1}\hat{Z}(k, s). \tag{55}$$

We have

$$Z(k+1) = (\Theta + MHKM^{-1})\hat{Z}(k, s) + \Theta\bar{V}(k). \tag{56}$$

Furthermore, the estimator sets

$$O(k+1, s) = (\Theta + MHKM^{-1})\hat{Z}(k, s) \tag{57}$$

and

$$Z(k+1, s) = (\Theta + MHKM^{-1})\hat{Z}(k, s). \tag{58}$$

Clearly, it follows that

$$\begin{aligned} |z_i(k+1) - o_i(k+1, s)| &\leq |\lambda_i|^{\alpha(k)+1} d_i(k-\alpha(k), s) \\ &> d_i(k+1, s). \end{aligned} \tag{59}$$

Clearly, the state prediction error will grow by $|\lambda_i|$.

At the time $k+1$, the encoder has access to $z_i(k+1)$ and finds that there exists $i \in \{1, 2, \cdots, n\}$ such that

$$|z_i(k+1) - z_i(k+1, e)| > d_i(k+1, e) \tag{60}$$

holds. This means that the encoder can know that no packet sent at the kth sampling interval was successfully delivered to the estimator at the time within $(kh, (k+1)h)$. Then, the encoder sets

$$\hat{z}_i(k, e) = o_i(k, e) \tag{61}$$

and obtains

$$O(k+1, e) = (\Theta + MHKM^{-1})\hat{Z}(k, e). \tag{62}$$

Then, it follows that

$$\begin{aligned} o_i(k+1, e) &= o_i(k+1, s), \\ d_i(k+1, e) &= d_i(k+1, s), \\ \hat{z}_i(k, e) &= \hat{z}_i(k, s). \end{aligned} \tag{63}$$

For this case, the encoder and the estimator can also work synchronously.

Combined with the arguments above, it follows that the encoder and the estimator can work synchronously. Then, we have

$$E|z_i(k) - o_i(k, s)| \leq \sum_{j=0}^{\infty} p_j |\lambda_i|^{\alpha(k)} d_i(k-\alpha(k), s), \tag{64}$$

where p_j denotes the probability of $\alpha(k) = j$. As k is large enough, it follows that

$$E|z_i(k) - o_i(k, s)| \leq \sum_{j=0}^{\infty} (1-p_e) p_d^j |\lambda_i|^j \phi_0. \tag{65}$$

If we assume that $p_d < \frac{1}{|\lambda_i|}$ holds, it follows that

$$E|z_i(k) - o_i(k, s)| < \frac{1-p_d}{1-p_d|\lambda_i|}\phi_0. \tag{66}$$

At the same time, it follows that

$$r(k) > (\alpha(k)+1) \sum_{i=1}^{n} \log_2 |\lambda_i| \quad \text{(bits/sample)}. \tag{67}$$

We take an expectation on it, and obtain

$$R > \sum_{j=0}^{\infty} p(\alpha(k) = j)(\alpha(k)+1) \sum_{i=1}^{n} \log_2 |\lambda_i| \quad \text{(bits/sample)}. \tag{68}$$

As k is large enough, it follows that

$$R > \frac{1}{1-p_e} \sum_{i=1}^{n} \log_2 |\lambda_i| \quad \text{(bits/sample)}. \tag{69}$$

Thus, the proof is complete. □

Remark 1. To deal with the state estimation problem, we employ the differential quantization method and give a new quantization coding scheme for System (1). The scheme is clearly different from the existing ones in [5] and [9]. It is shown in Theorem 1 that there exists a lower bound of data rate for observability. The paper [9] also gave a lower bound of data rate for observability. However, the paper [9] only consider the case without data packet dropout. The paper [5] gave the conditions for observability and controllability of linear systems subject to dada losses. We further consider the case with both data packet dropout and data rate limitation and give the sufficient condition for observability. Furthermore, in Theorem 1, we give an upper bound on the probability of packet dropout for observability (*i.e.*, $p_d < \frac{1}{|\lambda_i|}$), which is similar to that of [5]. The difference is that we further argue the bound under the data rate limitation, but the paper [5] considered the case with both packet dropout and delay.

5. Numerical Example

In this section, an numerical example is presented to illustrate the obtained result for the state estimation in System (1).

Consider System (1) with the following parameters

$$G = \begin{bmatrix} 12.67 & 22.12 \\ -4.38 & -7.11 \end{bmatrix}, H = \begin{bmatrix} 2.1 \\ -0.7 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \tag{70}$$

Let $x_1(0) \in [-5, 5]$ and $x_2(0) \in [-5, 5]$. We employ the feedback gain $k = [-6.0 \ \ -10.8]$. Let the sampling period $h = 0.5ms$. We may compute M and Θ, and obtain

$$M = \begin{bmatrix} 0.9273 & -0.8978 \\ -0.3743 & 0.4405 \end{bmatrix}, \Theta = \begin{bmatrix} 3.7425 & 0 \\ 0 & 1.8175 \end{bmatrix}. \tag{71}$$

First, we set the data rate of the channel and choose $R = 8.0$ (Kb/s). Furthermore, we set $p_d = 0.2$. Figure 2

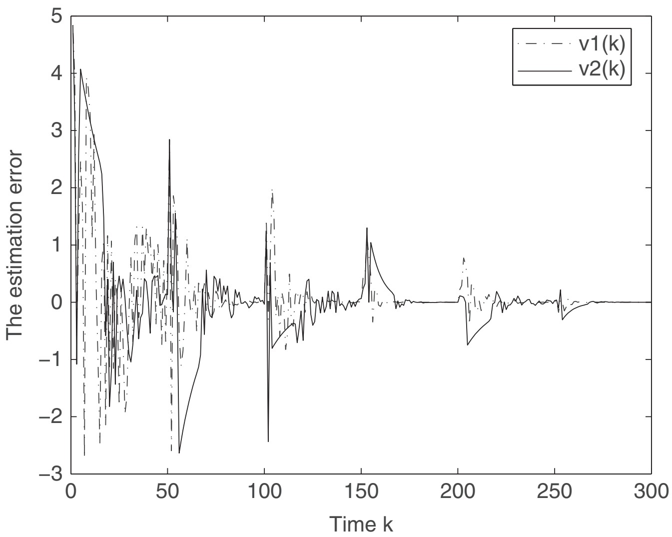

Figure 2. The estimation error with $p_d = 0.2$ and $R = 8.0$ (Kb/s).

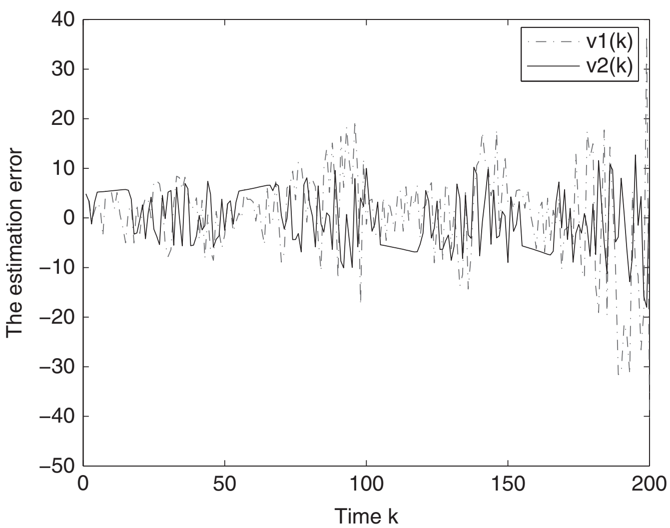

Figure 3. The estimation error with $p_d = 0.4$ and $R = 8.0$ (Kb/s).

shows the simulation result for this case. Clearly, it turns out that in this case, the system is observable.

Next, we enlarge the packet dropout probability and set $p_d = 0.4$. Furthermore, we still choose $R = 8.0$ (Kb/s). Figure 3 shows the simulation result for this case. Clearly, it turns out that in this case, the system is unobservable.

Now, we reduce the data rate and set $R = 4.0$ (Kb/s). Furthermore, we still set $p_d = 0.2$. Figure 4 shows the simulation result for this case. Clearly, it turns out that in this case, the system is unobservable.

6. Conclusion

In networked control systems, data packets on plant states are transmitted over a digital communication channel with both data packet dropout and data rate limitation. For this case, we proposed a new quantization coding scheme and derived sufficient conditions for observability. Here, we consider the case without data transmission delay. However, the quantization coding scheme above needs to be improved in the case with data transmission delay. For future research, we will consider this case and derive sufficient conditions for observability in this case. Furthermore, future work will seek to address these challenges in networked nonlinear control systems. An illustrative example was given to demonstrate the effectiveness of the proposed method.

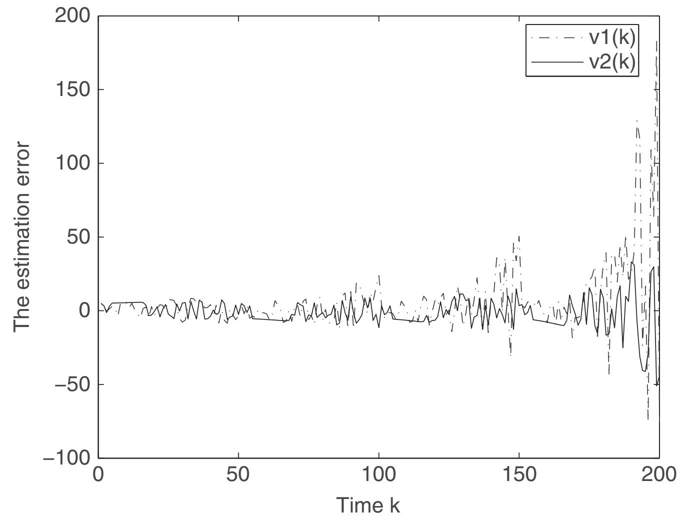

Figure 4. The estimation error with $p_d = 0.2$ and $R = 4.0$ (Kb/s).

Acknowledgement

This work is partially supported by the Fuzhou Key Science and Technology Special Projects (Grant: 2020FZZD0601) and the Fujian Province STS Projects (Grant: 2020T3008). The authors also gratefully acknowledge the helpful comments and suggestions of the reviewers, which have improved the presentation.

References

[1] Z. Zhou, H. Wang, Z. Hu, and X. Xue, Finite time stability of discrete markovian jump system over networks with random dual-delay, *Mechatronic Systems and Control*, *47*(3), 2019, 153–161.

[2] A. Cetinkaya, H. Ishii, and T. Hayakawa, Networked control under random and malicious packet losses, *IEEE Transactions on Automatic Control*, *62*(5), 2017, 2434–2449.

[3] P. Tallapragada, M. Franceschetti, and J. Cortés, Event-triggered second-moment stabilization of linear systems under packet drops, *IEEE Transactions on Automatic Control*, *63*(8), 2018, 2374–2388.

[4] R.M. Jungers, A. Kundu, and W.P.M.H. Heemels, Observability and controllability analysis of linear systems subject to dada losses, *IEEE Transactions on Automatic Control*, *63*(10), 2018, 3361–3376.

[5] Z. Ren, P. Cheng, L. Shi, and Y. Dai, State estimation over delayed mutihop network, *IEEE Transactions on Automatic Control*, *63*(10), 2018, 3545–3550.

[6] Q. Liu and R. Ding, State estimation for networked control systems with random time delays, *Mechatronic Systems and Control*, *46*(3), 2018, 115–120.

[7] Q. Liu, Minimum information rate for observability of linear systems with stochastic time delay, *International Journal of Control*, *92*(3), 2019, 476–488.

[8] Q. Liu. and C. Yu, Quantization and coding for networked control under data-rate limitations, *Mechatronic Systems and Control*, *48*(2), 2020, 128–133.

[9] S. Tatikonda and S.K. Mitter, Control under communication constraints, *IEEE Transactions on Automatic Control*, *49*(7), 2004, 1056–1068.

[10] Q. Liu and F. Jin, State estimation for networked control systems using fixed data rates, *International Journal of Systems Science*, *48*(9), 2017, 1818–1828.

[11] Q. Liu and D. Rui, Observability and stabilisability of networked control systems with limited data rates, *International Journal of Systems Science*, *49*(11), 2018, 2463–2476.

[12] Q. Ling, Bit rate conditions to stabilize a continuous-time scalar linear system based on event triggering, *IEEE Transactions on Automatic Control*, *62*(8), 2017, 4093–4100.

[13] L. Li, X. Wang, and M.D Lemmon, Efficiently attentive event-triggered systems with limited bandwidth, *IEEE Transactions on Automatic Control*, *62*(3), 2017, 1491–1497.

[14] M. Xia, V. Gupta and P.J. Antsaklis, Networked state estimation over a shared communication medium, *IEEE Transactions on Automatic Control*, *62*(4), 2017, 1729–1741.

[15] M. Muehlebach and S. Trimpe, Distributed event-based state estimation for networked systems: A LMI approach, *IEEE Transactions on Automatic Control*, *63*(1), 2018, 269–276.

[16] Y. Li, S. Philips, and R.G. Sanfelice, Robust distributed estimation for linear systems under intermittent information, *IEEE Transactions on Automatic Control*, *63*(4), 2018, 973–988.

[17] X. Ren, J. Wu, K.H. Johansson, G. Shi, and L. Shi, Infinite horizon optimal transmission power control for remote state estimation over fading channels, *IEEE Transactions on Automatic Control*, *63*(1), 2018, 85–100.

[18] X. Ren, J. Wu, K.H. Johansson, G. Shi, and L. Shi, Two nonlinear observers based sliding mode controller for a multivariable continuous stirred tank reactor, *Mechatronic Systems and Control*, *63*(1), 2018, 85–100.

[19] T. Cover and J. Thomas, *Elements of Information Theory.* New York: Wiley, 2006.

Biographies

Chunqiang Chen born in Fu'an, Fujian, China, is currently studying at the GNSS Research Center of Wuhan University. He is a senior engineer of the Virtual Manufacturing and Simulation Research Center of Haixi Research Institute of the Chinese Academy of Sciences. He is also deputy director of Beidou Open Laboratory (Quanzhou) and deputy director of Fujian Province Belt and Road Joint Laboratory.

Weiming Tang received the M.S. degree and Ph.D. degree from Wuhan University, Wuhan, China, in 2003 and in 2006, respectively. He is currently a professor in the GNSS Research Center of Wuhan University. He is mainly engaged in the teaching and scientific research of GNSS's high-precision real-time dynamic positioning algorithm research, application development, and system integration. He has about 70 papers.

Qingquan Liu received the M.S. degree and Ph.D. degree from Northeastern University, Shenyang, China, in 2008 and in 2012, respectively. In 2016, he became the head of Department Information Counter Teaching and Research of Shenyang Ligong University, Shenyang, China. His research interests lie in the intersection of communication, control, and information theory.

Zhihui Dang received a bachelor's degree in 2021 and graduated from Shenyang Ligong University. Now he is a graduate student at the School of Equipment Engineering, Shenyang Ligong University, majoring in detection, guidance and information countermeasure technology, researching sonar technology.