Proceedings of the Fifth IASTED International Conference
POWER AND ENERGY SYSTEMS
June 15-17, 2005, Benalmádena, Spain

VOLTAGE STABILITY OF WIND POWER SYSTEMS USING BIFURCATION ANALYSIS

S.A. El-Kashlan
Elect. & Control Dept.
AAST
Alexandria, Egypt
shosh97@mail.com

M. Abdel-Rahman
Elect. Power & Machines Dept.
Ain-Shams University
Cairo, Egypt
m_abdel_rahmanca@yahoo.com

H. El-Desouki
Elect. & Control Dept.
AAST
Alexandria, Egypt
hdesouki@eng.aast.edu

M.M. Mansour
Elect. Power & Machines Dept.
Ain-Shams University
Cairo, Egypt
mmsmansour@ieee.org

ABSTRACT
This paper deals with the voltage stability analysis of wind power systems subjected to gradual parameter variation, such as load variation using bifurcation theory, in which the system is studied by the eigenvalues of the linearized system differential-algebraic equations (DAE). It is shown using a single induction generator connected to the grid how the loadability curves (nose curves) can characterize the maximum load that can be installed before the voltage collapse occurs. This is an important contribution in the emerging field of wind generation connected to the main power system network, in particular its impact on the stability of the system due to the dynamic characteristics of wind generation.

KEY WORDS
Wind power systems, voltage stability, bifurcation.

Nomenclature

V_d and V_q: direct and quadrature axis components of induction generator terminal voltage.
E'_d and E'_q : real and imaginary components of induction generator voltage behind transient reactance.
r_s : stator resistance of induction generator.
I_d and I_q : direct and quadrature axis components of induction generator injected current.
X : induction generator open circuit reactance.
X' : induction generator open circuit transient reactance.
T'_0 : induction generator open circuit time constant (sec).
s : induction generator slip.
ω_0 : synchronous angular velocity.
H : inertia constant
P_m : input mechanical power.
P_e : electrical power output.

1. Introduction

Modern interconnected power networks are stressed, highly nonlinear and complex systems. Systems [1]. During the past few decades, power system engineers had serious issues imposed by increased power transfer over long distances. Difficulty in controlling bus voltages besides the non convergence of load flow calculations and small transient stability domain are just some of the problems routinely faced. An essential fact is that transmission systems operate well inside a linear domain but planning and operations must address power system nonlinear behavior. There is a need for analytical concepts and tools that work when nonlinearity is encountered [2]. Nowadays, some power systems start to face problems of integrating thousands of decentralized wind power megawatts spread over large extensions. Consequently the problems of planning, operation and control of power systems with large wind power have become very important issues [3]. In this paper, the focus is the wind power since it has already found an obvious potential to become a major source of energy. In general, integration of wind power systems raises different technical issues, viz., voltage stability and active/reactive power fluctuations. Most efforts have been dedicated to voltage quality analysis of wind power and to economics of power systems including wind power [3].
Wan and Brian in [4] pointed out the main factors for utility integration of solar and wind power. Studies from late 1970s until 1980s have provided a starting point and general classification of the most relevant power system aspects. One of the conclusions was that there were no clear limits on wind power integrations. Furthermore, penetration limits of wind energy were economic rather than technical. It also clearly concluded that spatial distribution of wind must be taken into account. A recent report by Nielsen, Varming and Gaardesatrup in [5] reviewed the technical options and constraints of integration of distributed power generation. Similar work was presented by Larsson in [6] where the focus was the economic aspects of large-scale wind power operation.
On the dynamic point of view, an extensive analysis of the potential impacts of wind turbines was developed on

the power quality but the work focused on small scale integration [7]. Similarly, Sørensen, Hansen, Janosi, J. Bech and Bak-Jensen in [8] have simulated the interaction between wind farms and power systems. However, Larsson in [6] characterized the modern wind turbines and explained how to assess the voltage quality for wind turbines using wind turbine characteristics. Moreover, [9] presented transient stability analysis of power system with large amount of wind power. The transient voltage collapse was investigated as well as the wind turbines behavior during short circuits.

It should be noted that dynamic simulations of large wind power systems are very expensive but most studies aggregate machines with similar dynamic characteristics. With the reduced model, the overall aggregate dynamic behavior can be easily investigated.

Voltage stability has been pointed out as another problem to large integration of wind power because wind farms demand reactive power. Taylor in [10] and Van Cutsem with Vournas in [11] presented an extensive explanation of the voltage stability problem and means of avoiding it. In addition, the Power System Stability Subcommittee of IEEE in [12] and Gao, Morison and Kundur in [13] suggested several tools to deal with that problem. Furthermore, DeMarco and Overbye in [14] reviewed the performance indices to predict proximity of voltage collapse as these indices could be use on-line or off-line to help operators to determine how close the system from collapse phenomenon is.

In this paper, the voltage stability problem of wind power systems is addressed in the context of bifurcation analysis. Section (2) is an overview of bifurcation analysis with some applications is introduced while in section (3), the mathematical background is highlighted on the formulation of nonlinear bifurcation problem. The case study associated with some technical considerations is presented in section (4) while the simulation results with the proposed model are obtained in section (5). Finally the recommended conclusion is addressed in section (6).

2. Bifurcation Analysis

A bifurcation point is that point in the parameter space at which a qualitative change in the system behavior is exhibited. In any nonlinear dynamic system, it is well known that the qualitative change in the behavior of the system with the associated variation of one or more parameters is due to bifurcations. In some cases, this parameter variation may result in a complicated behavior in which the system exhibits oscillatory behavior leading to instability [15].

Bifurcation analysis provides a means for studying dynamic mechanisms which may change structural stability of the system as some parameter varies slowly with time. Local bifurcations are readily evident in power systems as important reasons of instability. In one dimensional problems, the two most generic types of these are the saddle-node, Hopf and Singularity-Induced bifurcations as illustrated in Figure (1).

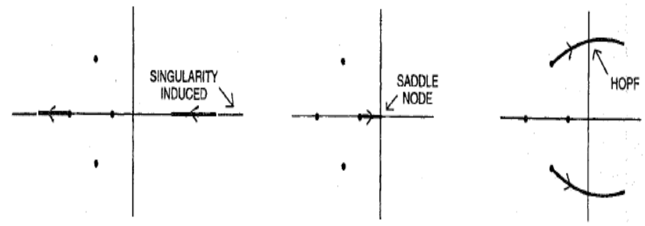

Figure 1. Locus of the linearized eigenvalues in s-plane

Saddle node bifurcation has become a widely accepted paradigm for one important form of voltage instability and linked to voltage collapse phenomenon. It is the common view in literature that the saddle node bifurcation happens with a bifurcation of equilibrium points during slow changes of parameters (real and reactive power injections in power systems). The saddle node bifurcation in the voltage collapse frame has been investigated in subsequent papers such as [16-19].

On the other hand, the importance of Hopf bifurcation has been increasingly recognized as it became clear that stability of the equilibrium could be lost by this mechanism before reaching the point of collapse. Hopf bifurcation occurs when a pair of complex conjugate eigenvalues moves from the left to right half of the complex plane crossing the imaginary axis at points other than the origin. There is substantial literature available on Hopf bifurcation in power systems and its relation to instability [20-26]. Hopf bifurcation concerns electromechanical oscillations, so it could arise due to variable net damping or frequency dependence of electrical torque and voltage control issues (AVRs) and typically triggered by system contingencies [27].

3. Mathematical Background

A power system dynamic model is characterized by a set of parameter dependent differential equations that are constrained by a set of algebraic equations. The differential-algebraic equations (DAE's) are:

$$\begin{aligned} \dot{x} &= f(x, y, \lambda) \\ 0 &= g(x, y, \lambda) \end{aligned} \tag{1}$$

where $x \in R^n$ is a vector of dynamic state variables, $y \in R^m$ is a vector of instantaneous state or algebraic variables (usually complex node voltages) while $\lambda \in R^k$ is any parameter of the system which changes slowly moving the system from one equilibrium point to another. The set of differential equations $f : R^{n+m+k} \rightarrow R^n$ represents the dynamics of the equipment (e.g. generators with their controls, FACTS devices and load at the buses). Whereas the set of algebraic equations $g : R^{n+m+k} \rightarrow R^m$ expresses the power flow at each bus. The x variables have their dynamics associated to y variables which are assumed to be so fast such that the constraints $g = 0$ are always satisfied. It should be pointed out that the variables y are variables which change instantaneously with variations of the x states. Therefore, the network power flows are modeled to be instantaneous in the differential-algebraic equation (DAE) model.

For an arbitrary fixed parameter λ_e, the steady state equilibrium points $\boldsymbol{P_e}$ = (x_e, y_e, λ_e) are obtained by solving the set of nonlinear algebraic equations given by:

$$\begin{aligned} 0 &= f(x, y, \lambda_e) \\ 0 &= g(x, y, \lambda_e) \end{aligned} \tag{2}$$

The above equations are normally called load flow equations. Any equilibrium solution given by points $\boldsymbol{P}$ defines the region of equilibria of the power system. Consequently, equilibrium implies that the system is at rest, i.e. equation (2) is satisfied but does not imply stability. Equation (2) is solved as a function of control parameter λ by using a continuation method [28].

The stability of an isolated equilibrium point $\boldsymbol{P_e}$ = (x_e, y_e, λ_e) which defines a local power system stability region is obtained by linearzing equation (1) about a certain operating point and carrying out an eigenvalue analysis to the resulting system. This linearization is obtained by a Taylor series expansion of first degree in the neighborhood of the equilibrium point of interest. The resulting system is:

$$\begin{bmatrix} \Delta\dot{x} \\ 0 \end{bmatrix} = J \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} \tag{3}$$

where

$$J = \begin{bmatrix} \dfrac{\partial f}{\partial x} & \dfrac{\partial f}{\partial y} \\ \dfrac{\partial g}{\partial x} & \dfrac{\partial g}{\partial y} \end{bmatrix}_{|(x_e, y_e, \lambda_e)} = \begin{bmatrix} f_x & f_y \\ g_x & g_y \end{bmatrix} \tag{4}$$

Stability of an isolated equilibrium point depends mainly upon the eigenvalues of the Jacobian matrix $\boldsymbol{J}$. If all eigenvalues have negative real parts, then the corresponding equilibrium solution is called a stable equilibrium point, otherwise it is an unstable equilibrium point. Such classification is stated as: stable node, unstable node and saddle point where the eigenvalues are located at the origin [29].

4. Case Study Application

Since system instability generally emerges from two types of events: gradual parameter variation, such as load variation and contingencies, therefore this work is focused on the voltage stability analysis of wind power systems subjected to gradual parameter variation, i.e. active power loading variation through the local bifurcation theory. The system is studied by the eigenvalues of the linearized system differential-algebraic equations (DAE).

To illustrate how stability index is determined via linearized eigenvalue analysis, a simple test system is proposed which consists of a single induction generator connected to a grid. It should be pointed out that the proposed model is similar to [30] but in this paper the synchronous generator is substituted with an induction generator. To compare static and dynamic bifurcation features and to obtain the necessary loading margin(s), a static load, be an R-L load, and a dynamic load, an induction motor load, are fed from the induction generator. Moreover, behavior of induction generator feeding an induction motor is of interest not only from the operational point of view but also from the view point of assessing its suitability to feed such loads. Figure (2) shows the single line diagram of a three-phase induction machine connected directly to the grid.

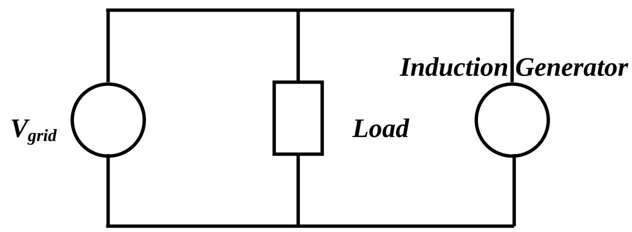

Figure 2. Schematic diagram for the proposed model

Induction generator Modeling:

Wind driven induction generator is an asynchronous machine with negative slip and torque. The voltage and torque equations for the induction machine rotating in synchronously rotating (*d-q*) reference frame are expressed in per unit values [31]:

Stator equations:

$$\begin{aligned} V_d &= E'_d + r_s\, I_d - X'\, I_q \\ V_q &= E'_q + r_s\, I_q + X'\, I_d \end{aligned} \tag{5}$$

Rotor equations:

$$\begin{aligned} \frac{d}{dt}(E'_d) &= \frac{1}{T'_0}[-E'_d - (X - X')\, I_q] + s\,\omega_0\, E'_q \\ \frac{d}{dt}(E'_q) &= \frac{1}{T'_0}[-E'_q - (X - X')\, I_d] - s\,\omega_0\, E'_d \end{aligned} \tag{6}$$

The mechanical equation governing the inertial dynamics of the rotor is given by:

$$\frac{P_m}{(1-s)} - P_e = -2H\,\frac{ds}{dt} \tag{7}$$

Electromagnetic torque equation is:

$$T_e = E'_d I_d + E'_q I_q \tag{8}$$

5. Numerical Simulation Results

Induction generator parameters of the 1.5 kW induction generator are presented in the Appendix.. For simplicity, induction motor rating is similar to that of the generator (i.e. slip is equal to 3 % for motor and -3 % for generator). All simulation results are obtained using PSAT software package developed by F. Milano [32].

A state-space linearized model is obtained using equations (5), (6) and (7). Table (1) shows the linearized eigenvalues for the overall system when subjected to a constant wind speed.

Load Type	Eigenvalues
Static Load (R-L)	$-0.05 \pm j1.95$ $-0.35 \pm j2.85$ $-0.79 \pm j3.57$ -2.69
Dynamic Load (Induction motor)	$-0.00 \pm j5.24$ $-0.15 \pm j6.33$ -1.75 -3.59 -5.82

Table. 1. Linearized eigenvalues of the overall system subjected to constant speed

As noticed from the above table that the static load fed from a constant speed induction generator does not cause any instability problem, but on the other hand the dynamic load does really affect it as a Hopf bifurcation is induced.

Stability Index:

The determination of a stability index is important to establish a distance from an equilibrium point to the closest bifurcation. Although this index can be expressed in terms of any controlled parameter space, it is usually expressed in terms of "loading margin." In this work, the loading margin (λ) is defined as a per unit loading factor from the base load to the maximum loading conditions.

$$\lambda = \frac{P_{base}}{P_{\max}} \tag{8}$$

Based on the above definition, Figure (3) represents the relation between loading margin and load phase voltage in static load (R-L) at constant speed while Figure (4) shows the same relation but in the case of dynamic load (induction motor).

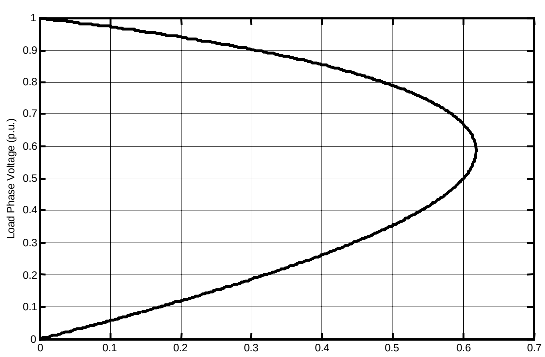

Figure 3. Loading margin (λ) in p.u. versus load phase voltage (p.u.) in case of R-L load

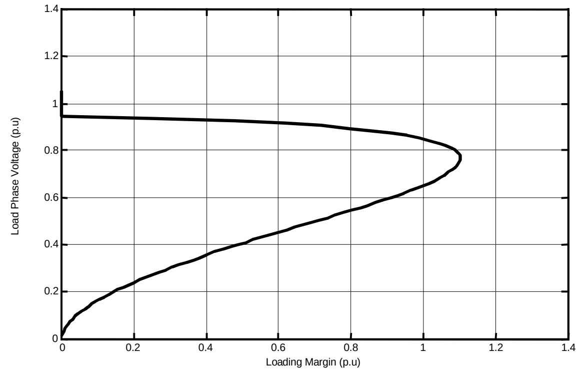

Figure 4. Loading margin (λ) in p.u. versus load phase voltage (p.u.) in case of induction motor load

In the case of R-L load, the maximum loading margin that can be achieved is 0.625 p.u. but there was no Hopf bifurcation indicating a stable operation. Analyzing the induction motor behavior, a dynamic Hopf bifurcation occurred at a loading margin λ^* = 2.37 p.u. which results in an oscillatory behavior in the voltage magnitude as illustrated in Figure (5).

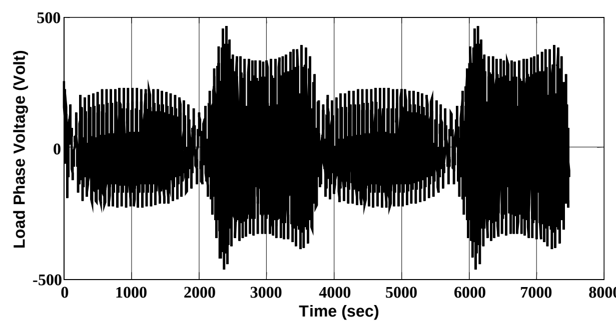

Figure 5. Load phase voltage (volt) versus time (sec)

6. Conclusion

In this paper, a wind power system model was proposed in context of bifurcation analysis. The model consisted of an induction generator feeding a static R-L load and a dynamic induction motor load. Simulation results have been applied on induction generator feeding a static R-L load and a dynamic induction motor load. Results have showed the loading margins and the bifurcation points. Voltage instability occurs when a induction motor load is connected to the case study system. However, for a static load voltage instability does not occur.

Appendix

Experimental measurements on a laboratory 1.5 kW, 127/220 V (line voltage), 6.3 / 3.2 A (line current), 50 Hz, 4-pole wound rotor induction machine parameters are:

(a) DC method for stator resistance:

$$r_s = \frac{V_{DC}}{I_{DC}} = \frac{9}{4.8} = 1.875\,\Omega$$

(b) Blocked-rotor test for stator and rotor reactances:

V_{Test} = 30 V, I_{Test} = 5.11 A, P_{Test} = 95.85 W

$$Z_{Test} = \frac{V_{Test}}{I_{Test}} = 5.8708\,\Omega,\quad R_{Test} = \frac{P_{Test}}{I_{Test}^2} = 3.6707\,\Omega$$

$$X_{Test} = \sqrt{Z_{Test}^2 - R_{Test}^2} = 4.5848\,\Omega,\quad X_s = X_r = 0.5\ X_{Test} = 2.29\Omega$$

Inertia constant (H) = 3 sec.

Acknowledgment

The authors would like to thank Dr. Federico Milano for his PSAT software package which helped to simulate the proposed wind power system.

References

[1] M. Wildberger, Stability and Nonlinear Dynamics in Power Systems, *IEEE Power Engineering Review*, *14* (10), 1994, 16-18.

[2] I.A. Hiskens, Analysis Tools for Power Systems-Contending with Nonlinearities, *Proc. of IEEE*, *83*(11), 1995, 1573-1587.

[3] P. Rosas, *Dynamic Influences of Wind Power on the Power System*, PhD Thesis, Technical University of Denmark, March 2003.

[4] Y. Wan and B.K. Parsons, "Factors Relevant to Utility Integration of Intermittent Renewable Technologies," NERL Report NREL/TP-463-4953, Colorado, USA, August 1993.

[5] J.E. Nielsen, S. Varming and C. Gaardesatrup, Review of Technical Options and Constraints For Integration of Distributed Generation in Electricity Networks, Report SUSTELNET,(on-line:www.sustelnet.net), Netherlands, December 2002.

[6] A. Larsson, *The Power Quality of Wind Turbines*, PhD dissertation, Department of Electrical Power Engineering, Chalmers University of Technology, Goteborg, Sweden, 2000.

[7] J.O. Tande, Applying Power Quality Characteristics of Wind Turbines for Assessing Impact on Voltage Quality, *Wind Energy Journal*, (5), 2002.

[8] P. Sørensen, A. Hansen, L. Janosi, J. Bech and B. Bak-Jensen, "Simulation of Interaction between Wind Farm and Power System," Riso Report No. 1281, Roskilde, Denmark, December 2001.

[9] V. Akhmatov, A.H. Nielsen and H. Knudsen, Electromechanical interaction and stability of power grids with windmills, *Proc. of IASTED International Conference on Power & Energy Systems*, Marbela, Spain, September 2000.

[10] C.W. Taylor, *Power System Voltage Stability* (New York, McGraw-Hill Inc., 1994).

[11] T. Van Cutsem and C. Vournas, *Voltage Stability of Electric Power Systems* (London, Kluwer Academic Publishers, 1998).

[12] P. Kundur, J. Paserba, V. Ajjaarapu, G. Anderson, A. Bose, C. Canizares, N. Hatziargyriou, D.J. Hill, A. Stankovic, C.W. Taylor, T.Van Cutsem and V.Vittal, Definitions and Classification of Power System Stability, *IEEE Trans. Power Syst.*, *19*(3), 2004, 1387-1401.

[13] B. Gao, G.K. Morison and P. Kundur, Towards the Development of a Systematic Approach for Voltage Stability Assessment of Large-Scale Power Systems, *IEEE Trans Power Syst.*, *11*(3), 1996, 1314-1324.

[14] C. DeMarco and T. Overbye, An Energy based security Measure for Assessing Vulnerability To Voltage Collapse, *IEEE Trans. Power Systems*, *5*(2), 1990, 419-427.

[15] J. Guckenheimer and P. Holmes, *Nonlinear Oscillations, Dynamical Systems and Bifurcation of Vector Fields* (New York: Applied Mathematical Sciences, Springer-Verlag, 1986).

[16] I. Dobson and L. Lu, Computing an optimum direction in control space to avoid saddle node bifurcation and voltage collapse in electric power systems, *IEEE Trans. Automat. Control*, *37*(10), 1992, 1616– 1620.

[17] S. Greene, I. Dobson, and F. L. Alvarado, Sensitivity of the loading margin to voltage collapse with respect to arbitrary parameters, *IEEE Trans. Power Syst.*, *12*(1), 1997, 262–272.

[18] A. A. P. Lerm and A. S. e Silva, The use of voltage profile on oscillatory instability control, *Proc. 33rd North Amer. Power Symp.*, Waterloo, ON, Canada, Oct. 2000.

[19] V. Ajjarapu and B. Lee, Bifurcation theory and its application to nonlinear dynamical phenomena in an electrical power system, *IEEE Trans. Power Syst.*, *7*(1), 1992, 424–431.

[20] C. D. Vournas, M. A. Pai, and P. W. Sauer, The effect of automatic voltage regulation on the bifurcation evolution in power systems, *IEEE Trans. Power Syst.*, *11*(4), 1996, 1683–1688.

[21] K. Kim, H. Schattler, V. Venkatasubramanian, J. Zaborszky, and P.Hirsch, Methods for calculating oscillations in large power systems, *IEEE Trans. Power Syst.*, *12*(4), 1997, 1639–1648.

[22] I. A. Ozcan and H. Schattler, On the calculation of the feasibility boundary for differential-algebraic systems, *Proc. IEEE Conf. on Decision and Control*, Phoenix, AZ, 1999, 2580–2586.

[23] A. I. Zecevic and D. M. Miljkovic, The effects of generation redispatch on Hopf bifurcations in electric power systems, *IEEE Trans. Circuits Syst. I, 49*(8), 2002, 1180–1186.

[24] I. Dobson, F. Alvarado, and C. L. DeMarco, Sensitivity of Hopf bifurcations to power system parameters, *Proc. 31st Conf. on Decision and Control*, Tucson, AZ, 1992, 2928–2933.

[25] E.H. Abed and J.H. Fu, Local Feedback Stabilization and Bifurcation Control; I. Hopf Bifurcation, *Systems and Control Letters*, *7*(1), 1986, 11-17.

[26] E.H. Abed and J.H. Fu, Local Feedback Stabilization and Bifurcation Control; II. Stationary Bifurcation, *Systems and Control Letters*, *8*(2), 1987, 467-473.

[27] N. Mithulananthan, *Hopf Bifurcation Control and Indices for Power System with Interacting Generator and*

FACTS Controller, PhD Thesis, University of Waterloo, Ontario, Canada, 2002.
[28] S. Ayasun, C. O. Nwankpa, and H. G. Kwatny, Bifurcation and singularity analysis with voltage stability toolbox, *Proc. of 31st North American Power Symposium (NAPS)*, California Polytechnic State University, San Luis Obispo, CA, 1999, 390-397.
[29] R. Seydel, *Practical Bifurcation and Stability Analysis-From Equilibrium to Chaos* (New York: Springer-Verlag, 1994).
[30] I. Dobson, H-D. Chiang, J.S. Thorp and L. Fekih-Ahmed, A Model of Voltage Collapse in Electric Power Systems, *Proc. 27th Conf. on Decision and Control*, Austin, Texas, 1988, 2104-2109.
[31] P. Krause, *Analysis of Electric Machinery* (New York, McGraw-Hill Inc., 1986).
[32] F. Milano, *Power System Analysis Toolbox Manual*, Version 1.3.3, January 2005.